Name Key

Date _____ Per _____

Algebra II
WS 8.1 Word Probs. Variations

Show the work to solve each of the following word problems. Be sure to include units in your answers.

1. The number of kilograms of water in a person's body varies directly as the person's mass. A person with a mass of 90 kg contains 60 kg of water. How many kilograms of water are in a person with a mass of 50 kg?

$kg = mk$ $60 = K(90)$ $K = 2/3$ $Kg = \frac{2}{3}(50)$ $Kg = \frac{100}{3}$

2. The amount of fertilizer needed for a lawn varies directly with the area of the lawn. If 4 pounds of fertilizer are needed for 500 square feet of lawn, how much is needed for Dr. Quagmire's lawn, which is rectangular in shape and measures 25 feet by 50 feet?

$F = KA$ $A = 25 \cdot 50 = 1250 ft^2$ $4 = K(500)$ $K = \frac{4}{500} = \frac{2}{250} = \frac{1}{125}$ $F = \frac{1}{125}(1250)$ **F = 10 lbs**

3. The time, t, it takes to travel a certain distance varies inversely as the speed, s. Write an equation if $t = 10$ hours when $s = 80$ km/hr.

$t = \frac{K}{s}$ $10 = \frac{K}{80}$ $800 = K$ $t = \frac{800}{s}$

4. The volume of gas varies inversely as the pressure. A helium-filled balloon has a volume of 21 m^3 at sea level, where the pressure is 1 atmosphere. The balloon rises to an altitude where the pressure is 0.7 atmospheres. What is the volume?

$V = \frac{K}{P}$ $21 = \frac{K}{1}$ $K = 21$ $V = \frac{21}{.7}$ **$V = 30\ m^3$**

5. For piano wires under the same tension, the # of vibrations per second (frequency) of each wire is inversely proportional to the length of the wire. A wire 0.75 m long vibrates 480 times per second. How long is a wire that vibrates 300 times per second?

$F = \frac{K}{L}$ $480 = \frac{K}{.75}$ $K = 360$ $300 = \frac{360}{L}$ $300L = 360$ **L = 1.2m**

6. The distance, d, that a free-falling body falls varies directly as the square of the time, t, that it falls. If $d = 36$m when $t = 3$ sec, find the value of the variation constant, k. Then find d when $t = 5$ sec.

$d = kt^2$ $36 = K(3)^2$ $36 = K(9)$ $K = 4$ $d = 4(5)^2$ $d = 4(25) =$ **100 m**

7. The price, p, of a pizza varies directly as the square of its radius, r. If $p = \$6.00$ when $r = 10$ cm, find the value of k. Then find p when $r = 15$ cm.

$P = k(r)^2$ $6 = K(10)^2$ $6 = K(100)$ $K = 6/100 = 3/50$ $P = \frac{3}{50}(15)^2$ $P = \frac{3}{50}(225) =$ **$13.50**

8. The brightness of illumination, I, of an object varies inversely as the square of its distance, d, from the source of the illumination. If $I = 18$ luxes when $d = 4$ m, find the value of k. Then find I when $d = 3$m.

$I = \frac{K}{d^2}$ $18 = \frac{K}{(4)^2}$ $K = 288$ $I = \frac{288}{3^2} = \frac{288}{9} =$ **32 luxes**

For 10 – 17, translate each statement into a formula, using k as the constant of variation.

10. V varies jointly as B and h. $V = Bh$

11. t varies directly as W and inversely as n. $t = \frac{Wk}{n}$

12. h varies directly as W and inversely as the square of r. $h = \frac{Wk}{r^2}$

13. I varies jointly as A and H and inversely as T. $I = \frac{AHk}{T}$

14. The volume, V, of a gas varies directly as the temperature, T, and inversely as the pressure, P. $V = \frac{Tk}{P}$

15. The collision impact, I, of an automobile varies jointly as the mass, m, and the square of the speed, s. $I = kms^2$

16. The safe load, s, for a beam, varies jointly as the breadth, b, and the square of the depth, d, and inversely as the length, l, between supports. $S = \frac{kbd^2}{l}$

17. The gravitational force, g, between two objects varies jointly as the mass of the first, m_1, and the mass of the second, m_2, and inversely as the square of the distance, d, between them. $g = \frac{km_1m_2}{d^2}$

18. The heat loss through a glass window *varies jointly* as the area of the window and the difference between the inside and outside temperatures. If the loss through a window with area 3 m^2 is 720 BTU when the temperature difference is 15 C, what is the heat loss through a window with area 4.5 m^2 when the temperature difference is 12 C?

$LOSS = KA \cdot (inside - outside)$ $720 = K(3)(15)$ $720 = 45K$ $\boxed{K = 16}$

$LOSS = 16(4.5)(12)$ $LOSS = 864$ BTU

19. The conductance of a wire *varies directly* as the square of the wire's diameter and *inversely* as its length. Fifty meters of wire with diameter 2 mm has conductance 0.12 mho (mho, which is ohm spelled backwards, is a unit of conductance). If a wire of the same material has length 75 m and diameter 2.5 mm, what is its conductance?

$C = \frac{Kd^2}{L}$ $.12 = \frac{K(2)^2}{50}$ $6 = K(4)$ $K = 1.5$ $C = \frac{1.5(2.5)^2}{75}$ $C = .125$ mho

20. The volume of a cone *varies jointly* as the height and the square of the radius of the base. A cone of height 8 cm and base diameter 9 cm has a volume 54π. Find the constant of variation and a general formula for the volume of a cone. Write both the constant and the equation in terms of π.

radius 4.5

$V = Khr^2$ $54\pi = K(8)(4.5)^2$ $K = \frac{54\pi}{162} = \boxed{\frac{\pi}{3}}$ $\boxed{V = \frac{\pi}{3}hr^2}$

21. The stretch in a wire under a given tension *varies directly* as the length of the wire and *inversely* as the square of its diameter. A wire having length 2 m and diameter 1.5 mm stretches 1.2 mm. If a second wire of the same material (and under the same tension) has length 3 m and diameter 2.0 mm, find the amount of stretch.

$S = \frac{kL}{d^2}$ $1.2 = \frac{k(2)}{(1.5)^2}$ $1.2 = \frac{2k}{2.25}$ $K = 1.35$ $S = \frac{1.35(3)}{(2)^2} = 1.0125$ mm

Mixed answers for #18-21: 0.125; 1.0125; $\frac{\pi}{3}$; $V = \frac{\pi}{3}hr^2$; 864