Algebra 2
Ch. 2.1

Name: Key

Objective: To graph relations and identify functions.

Section 2.1 – Relations and Functions

A relation is a set of pairs of input and output values. You can represent each relation in 4 different ways:

Ordered Pairs

(input, output)

(x, y)

(–3, 4)

(3, –1)

(4, –1)

(4, 3)

Mapping Diagram

Input Output

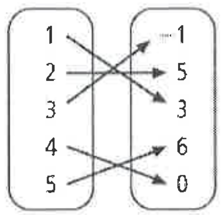
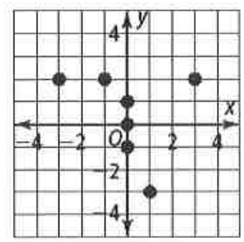


Table of Values

x Input	y Output
–3	4
3	–1
4	–1
4	3

Graph



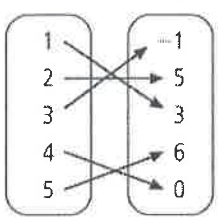
The Domain of a relation is the set of inputs, also called x-coordinates of the ordered pairs. The range is the set of outputs, also called the y-coordinates, of the ordered pairs.

A Function is a relation in which each element of the Domain corresponds with exactly one element of the range.

Examples:

Determine whether each relation is a function. List the domain and range of each relation.

1. Domain Range



Function

D = {1, 2, 3, 4, 5}

R = {-1, 3, 5, 6, 0}

2. {(1, 4), (3, 2), (5, 2), (1, –8), (6, 7)}

Not a function

D = {1, 3, 5, 6}

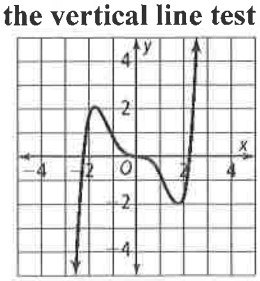
R = {4, 2, -8, 7}

You can use the vertical line test to determine whether a relation is a function. The vertical line test states that if a vertical line passes through more than 1 point on the graph of a relation, then the relation is not a function.

Examples:

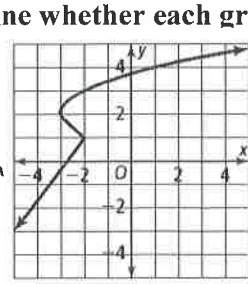
Use the vertical line test to determine whether each graph represents a function.

3.



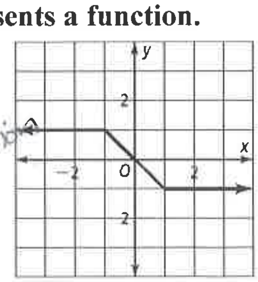
Function

4.



Not a function

5.



Function

Function Notation:

Equations that represent functions are often written in Function Notation. A Function rule is an equation that represents an output value in terms of an input value. (y in terms of x)
In the equation $y = 3x + 2$, we let $y = f(x)$ so the equation can now be written as $f(x) = 3x + 2$. The symbol, $f(x)$, is read f of x. The independent variable, x, represents the input of the function. The dependent variable, $f(x)$, represents the output of the function. It is called the dependent variable because its value depends on the input value.

Examples:

Evaluate each function for the given value of *x*, and write the input *x* and the output *f* (*x*) as an ordered pair.

6. $f(x) = -3x + 2$ for $x = 3$

$f(3) = -3(3) + 2$
$= -9 + 2$
$= -7$
$(x, f(x)) = (3, -7)$

7. $f(x) = \frac{1}{2}x - 1$ for $x = -2$

$f(-2) = \frac{1}{2}(-2) - 1$
$= -1 - 1$
$= -2$
$(x, f(x)) = (-2, -2)$

Write a function rule to model the cost of renting a truck for one day. Then evaluate the function for the given number of miles.

8. Daily rental: \$19.95 cost = rent + per mile

Rate per mile: \$.50 per mile

Miles traveled: 73 miles

$f(m) = \$19.95 + \$.50(m)$
$f(73) = \$19.95 + .50(73)$
$= \$56.45$

The truck will cost \$56.45

9. Daily rental: \$39.95

Rate per mile: \$.60 per mile

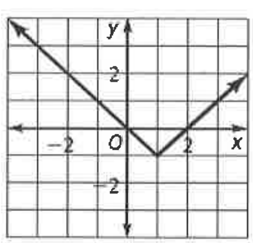
Miles traveled: 48 miles

$f(m) = \$39.95 + \$.60(m)$
$f(48) = \$39.95 + \$.60(48)$
$= \$68.75$

The truck will cost \$68.75

Find the domain and range of each relation, and determine whether it is a function.

10.

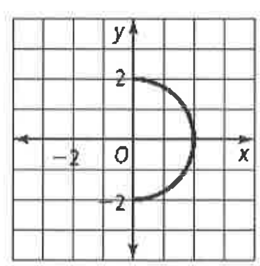


Function

$D = \{\text{all } \mathbb{R} \text{ \#'s}\}$

$R = \{y \mid y \geq -1\}$

11.



Not a Function

$D = \{x \mid 0 \leq x \leq 2\}$

$R = \{y \mid -2 \leq y \leq 2\}$

Section 2.2 – Direct Variation

Objective: To write and interpret direct variation equations.

You can write a formula for a direct variation function as $y = kx$ or $\frac{y}{x} = k$, where $k \neq 0$. x represents input values, and y represents output values. The formula $\frac{y}{x} = k$ says that, except for $(0,0)$ the ratio of all output - input pairs equals the constant k, the constant of variation.

Examples:

For each function, determine whether y varies directly with x. If so, find the constant of variation and write the function rule. To start, write ratios of output to input.

1.

x	y
3	–9
4	–12
5	–15

$\frac{y}{x} = \frac{-9}{3} = \frac{-12}{4} = \frac{-15}{5}$

$\frac{y}{x} = -3$ ∴ y varies directly with x

∴ constant variation $= -3$

$\boxed{y = -3x}$

2.

x	y
2	6
5	10
10	30

$\frac{y}{x} = \frac{6}{2} \times \frac{10}{5} \times \frac{30}{10}$

NO

∴ $\frac{y}{x}$ is not constant

y does not vary directly with x

3.

x	y
–4	–8
1	2
3	6

$\frac{y}{x} = \frac{-8}{-4} = \frac{2}{1} = \frac{6}{3}$

$\frac{y}{x} = 2$ y varies directly with x

∴ Constant variation $= 2$

Determine whether y varies directly with x. If so, find the constant of the variation.

4. $y = 5x$

y varies directly with x

$k = 5$

5. $\frac{3y}{3} = \frac{4x}{3} + \frac{6}{3}$

$y = \frac{4}{3}x + 2$

y doesn't vary directly

$y = kx$

6. $y = \frac{7}{x}$

$y = kx$

Doesn't vary directly

7. $y = -\frac{2}{3}x$

y does vary directly with x

$k = -\frac{2}{3}$

For Exercises 8–10, y varies directly with x.

8. If $y = -2$ when $x = 1$, find x when $y = 4$.

$y = kx$

$-2 = k(1)$

$k = -2$

$y = -2x$

$y = 4$; $\frac{4}{-2} = \frac{-2x}{-2}$

$\boxed{-2 = x}$

9. If $y = 4$ when $x = 5$, find y when $x = 10$.

$y = kx$

$4 = k(5)$

$k = \frac{4}{5}$

$y = \frac{4}{5}x$

$y = \frac{4}{5}(10)$

$\boxed{y = 8}$

Two Methods

$\frac{4}{5} = \frac{y}{10}$

$5y = 40$

$\boxed{y = 8}$

10. If $y = 12$ when $x = 36$, find x when $y = 7$.

$y = kx$

$12 = k(36)$

$k = \frac{1}{3}$

$y = \frac{1}{3}x$

$7 = \frac{1}{3}x$

$\boxed{21 = x}$

11. The length of an object's shadow varies directly with the height of the object. A 15 ft tree casts a 60 ft shadow.

a. Write a function rule and determine the constant of variation.
b. What length shadow would a 7 ft tree cast?
c. What height tree would cast a 90 ft long shadow?

a.) $y = kx$

object's shadow = k(height)

$\frac{60}{15} = \frac{15k}{15}$

$4 = k \rightarrow$ $\boxed{\text{object's shadow} = 4(\text{height})}$

b.) $y = 4(7)$

$\boxed{y = 28 \text{ ft}}$

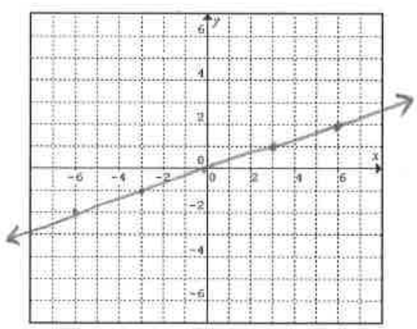
c.) $\frac{90}{4} = \frac{4x}{4}$

$\boxed{x = 22.5 \text{ ft}}$

Write and graph a direct variation that passes through each point.

12. (6, 2)

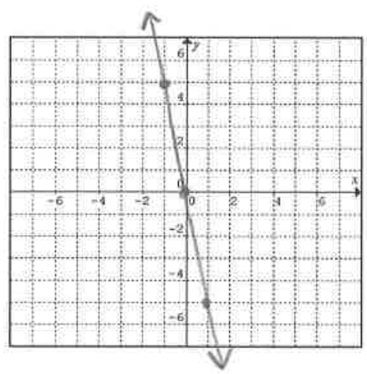
$\frac{y}{x} = \frac{2}{6} = \frac{1}{3}$



$y = \frac{1}{3}x$

13. (−1, 5)

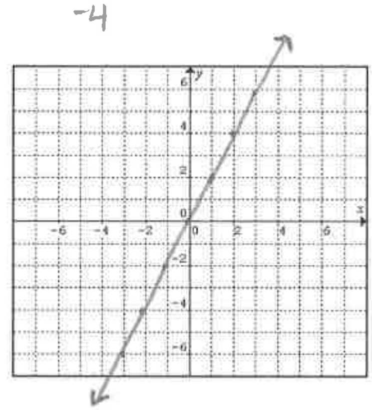
$\frac{5}{-1} = -5$



$y = -5x$

14. (−4, −8)

$\frac{-8}{-4} = 2$



$y = 2x$

Objective: To graph linear Equations. To write equations of Lines.

Sections 2.3 – Linear Functions and Slope-Intercept Form

Slope – The slope of a line through points (x_1, y_1) and (x_2, y_2) is the ratio of the vertical change to the corresponding horizontal change. Slope is denoted by the symbol m.

$$\text{Slope} = \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } x_2 - x_1 \neq 0$$

A function whose graph is a line is a linear function. You can represent linear function with a linear equation, such as $y = 6x - 4$.

A solution of a linear equation is any ordered pair (x, y) that makes the equation true.

A special form of a linear equation is called slope-intercept form, written as $y = mx + b$, where m is the slope of the line and b is the *y*-intercept.

An *intercept* of a line is a point where a line crosses an axis. The y-intercept of a nonvertical line is the point at which the line crosses the y-axis. The x-intercept of a nonhorizontal line is the point at which the line crosses the x-axis.

Examples:

Find the slope of the line through each pair of points.

1. $\overset{x_1}{(-3}, \overset{y_1}{-2)}$ and $\overset{x_2}{(1}, \overset{y_2}{6)}$ $\boxed{m = 2}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-2)}{1 - (-3)} = \frac{6 + 2}{1 + 3} = \frac{8}{4} = 2$$

2. $\left(\frac{1}{2}, \frac{2}{3}\right)$ and $\left(\frac{3}{2}, \frac{5}{3}\right)$

$$m = \frac{\frac{5}{3} - \frac{2}{3}}{\frac{3}{2} - \frac{1}{2}} = \frac{\frac{3}{3}}{\frac{2}{2}} = 1$$

$\boxed{m = 1}$

3. $(-3, -3)$ and $(-1, -3)$

$$m = \frac{-3 - (-3)}{-1 - (-3)} = \frac{-3 + 3}{-1 + 3} = \frac{0}{2} = 0$$

4. $(4, -1)$ and $(-2, -3)$

$$m = \frac{-3 - (-1)}{-2 - (4)} = \frac{-3 + 1}{-2 - 4} = \frac{-2}{-6} = \frac{1}{3}$$

$\boxed{m = \frac{1}{3}}$

Write an equation for each line.

5. $m = -4$ and the *y*-intercept is 3.

$y = -4x + 3$

6. $m = \frac{2}{5}$ and the *y*-intercept is $\frac{17}{5}$.

$y = \frac{2}{5}x + \frac{17}{5}$

7. $m = 0$ and the *y*-intercept is -4.

$y = 0x - 4$

$y = -4$

Find the slope and *y*-intercept of each line.

8. $3x - 4y = 12$

$-3x \quad -3x$

$\frac{-4y}{-4} = \frac{-3x}{-4} + \frac{12}{-4}$

$y = \frac{3}{4}x - 3$

$\boxed{m = \frac{3}{4}}$

$\boxed{\text{yint} = (0,-3)}$

9. $y = -2$

$\boxed{m = 0}$

$\boxed{\text{yint} = -2 \text{ or } (0,-2)}$

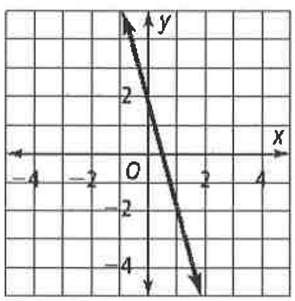
10. $f(x) = \frac{5}{4}x + 7$

$\boxed{m = \frac{5}{4}}$ $\boxed{\text{yint} = (0,7)}$

11. $x = 5$

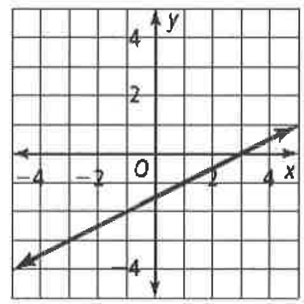
$\boxed{m = \text{undefined} \quad \text{yint} = \text{none}}$

12.



$\boxed{m = -4}$ $\boxed{\text{yint} = (0,2)}$

13.



$\boxed{m = \frac{1}{2}}$

$y = \frac{1}{2}x + b$

$0 = \frac{1}{2}(3) + b$

$0 = \frac{3}{2} + b$

$b = -\frac{3}{2} = -1.5$

$\boxed{\text{yint} = (0,-1.5)}$

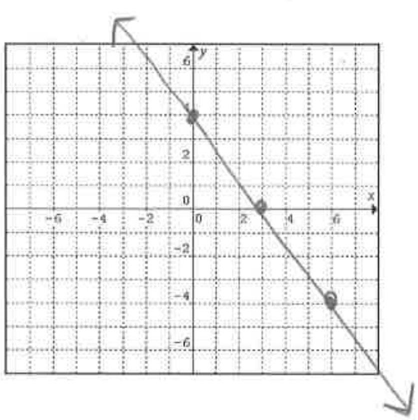
Graph each equation.

14. $4x + 3y = 12$

$-4x \quad -4x$

$\frac{3y}{3} = \frac{-4x}{3} + \frac{12}{3}$

$\boxed{y = -\frac{4}{3}x + 4}$



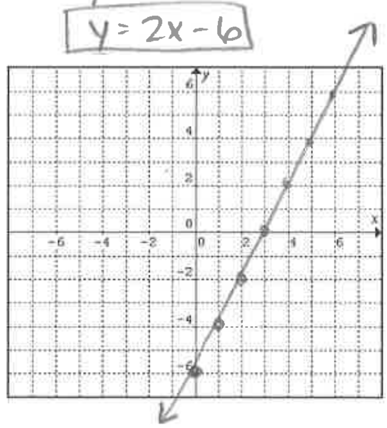
15. $\left(\frac{x}{3} - \frac{y}{6} = 1\right) \cdot 6$

$2x - y = 6$

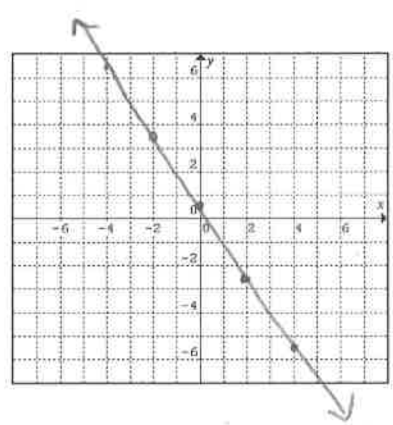
$-2x \quad -2x$

$-y = -2x + 6$

$\boxed{y = 2x - 6}$



16. $y = -\frac{3}{2}x + \frac{1}{2}$



Sections 2.4 – More About Linear Equations

positive

Standard form of a Line: $Ax + By = C$, where *A*, *B*, and *C* are integers and $A > 0$.

Slope-Intercept: $y = mx + b$, where *m* is the slope and *b* is the y-int.

To graph a line in *standard form,* use the x and y - intercepts. (x,0) (y,0).

Sketch the graph using the intercepts.

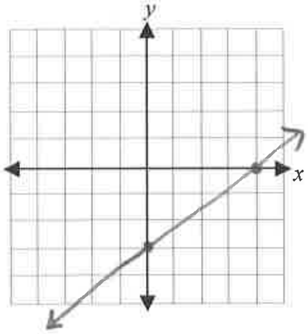
1. $3x - 4y = 12$

$3(0) - 4y = 12$
$-4y = 12$
$y = -3$

$3x - 4(0) = 12$
$3x = 12$
$x = 4$

x	*y*
0	-3
4	0

$m = 3/4$



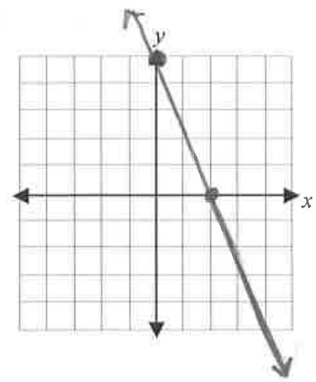
2. $5x + 2y = 10$

$5(0) + 2y = 10$
$2y = 10$
$y = 5$

$5x + 2(0) = 10$
$5x = 10$
$x = 2$

x	y
0	5
2	0

$m = -5/2$



More with Slope

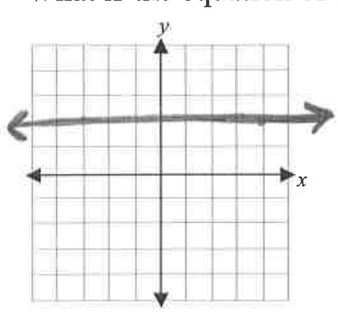
A line has a slope of -3. If two points on the line are $(\overset{x_1}{1}, \overset{y_1}{5c+9})$ *and* $(\overset{x_2}{7}, \overset{y_2}{2c})$, find c.

$\dfrac{2c - (5c+9)}{7-1} = -3 \qquad \dfrac{2c - 5c - 9}{6} = -3 \qquad 2c - 5c - 9 = -18$

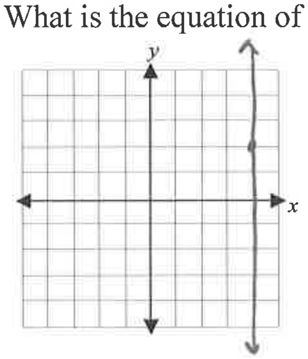
$-3c = -9$

$\boxed{c = 3}$

Graph the *horizontal line* through the point $(4, 2)$.
What is the equation of the line? $y = 2$



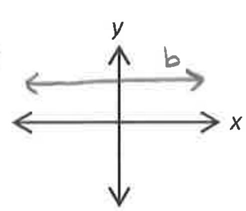
Graph the *vertical line* through the point $(4, 2)$.
What is the equation of the line? $x = 2$



Horizontal Lines

Eq: $y = b$

Graph:



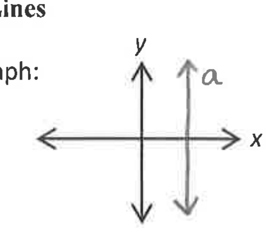
Slope: $m = 0$

x-axis → y = 0

Vertical Lines

Eq: $x = a$

Graph:



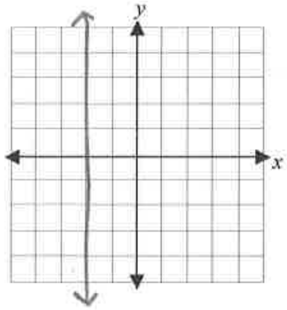
Slope: undefined

y axis → x = 0

Graph the lines and give the slope.

1. $x = -3$

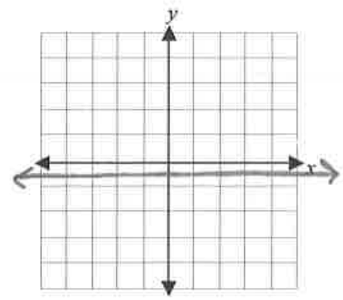
m=undefined



2. $-8y = 4$

$y = -\frac{1}{2}$

m=0



Given the slope and y-intercept, you can write the equation of a line in slope-intercept form. You can also write the equation of a line in Point-Slope form.

Point-Slope Form of a Line

Eq: $y - y_1 = m(x - x_1)$

where (x_1, y_1) = Point on line and m = slope

use for writing equation of a line

*Note: Leave answers in Point Slope Form unless otherwise

Use the *point-slope* form to write the *slope-intercept* and *standard* form of the line

1. with slope $\frac{15}{2}$ passing through $(3, -6)$.

$y - -6 = \frac{15}{2}(x - 3)$

$(y + 6 = \frac{15}{2}x - \frac{45}{2})2$

$2y + 12 = 15x - 45$

$\boxed{15x - 2y = 57}$

$\frac{-2y}{-2} = \frac{-15x}{-2} + \frac{57}{-2} \rightarrow \boxed{y = \frac{15}{2}x - \frac{57}{2}}$

2. passing through the points $(5, -5)$ and $(2, -3)$.

$m = \frac{-3 - -5}{2 - 5} = \frac{-3 + 5}{-3} = -\frac{2}{3}$

$y - -5 = -\frac{2}{3}(x - 5)$

$(y + 5 = -\frac{2}{3}x + \frac{10}{3})3$

$3y + 15 = -2x + 10$

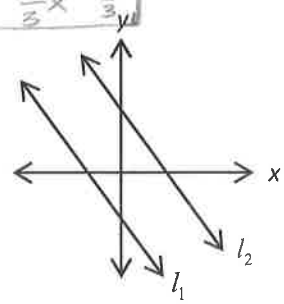
$\boxed{2x + 3y = -5}$

$3y = -2x - 5$

$\boxed{y = -\frac{2}{3}x - \frac{5}{3}}$

Parallel Lines

Two lines are parallel ⇔ they have the same slope.



Determine whether the two lines are ||.

1. $2x + 4y = -3$ and $6y = -3x + 5$

$4y = -2x - 3$

$y = -\frac{1}{2}x - \frac{3}{4}$

$y = -\frac{1}{2}x + \frac{5}{6}$

$m = -\frac{1}{2}$

$m = -\frac{1}{2}$

$\boxed{\text{parallel}}$

2. $5y - 2x = 7$ and $-3x + 4y = 25 + 7x$

$5y = 2x + 7$

$y = \frac{2}{5}x + \frac{7}{5}$

$\frac{4y}{4} = \frac{10x}{4} + \frac{25}{4}$

$y = \frac{5}{2}x + \frac{25}{4}$

$m = \frac{2}{5}$

$m = \frac{5}{2}$

$\boxed{\text{not parallel}}$

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3. Using the *point-slope form*, write the *slope-intercept* of the line that passes through $(-6,5)$ and is is parallel to $2x+3y=-3$.

$$\begin{aligned} 2x+3y&=-3 \\ -2x\quad &\quad -2x \\ 3y&=-2x-3 \\ y&=-\tfrac{2}{3}x-1 \\ m&=-\tfrac{2}{3} \end{aligned}$$

$y-5=-\frac{2}{3}(x+6)$

$y-5=-\frac{2}{3}x-4$

$\boxed{y=-\frac{2}{3}x+1}$

4. Using the *point-slope form*, write the *form* of the line that passes through $(-3,-1)$ and parallel to $4y-11x=12$.

$4y=11x+12$

$y=\frac{11}{4}x+3$

$m=\frac{11}{4}$

$y+1=\frac{11}{4}(x+3)$

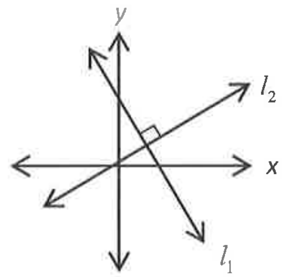
$\left(y+1=\frac{11}{4}x+\frac{33}{4}\right)4$

$4y+4=11x+33$

$4y=11x+29$

$4y-11x=29$

$\boxed{11x-4y=-29}$



Perpendicular Lines

Two nonvertical lines are perpendicular $\Leftrightarrow$ the slopes are

opposite reciprocal

Determine whether the two lines are $\perp$.

1. $8x=6y+5$ and $\frac{1}{2}x-\frac{2}{3}y=5$

$8x-6y=5$

$m=\frac{8}{6}=\frac{4}{3}$

$m=\frac{1}{2}\cdot\frac{3}{2}=\frac{3}{4}$

$\boxed{\text{not } \perp}$

2. $-4x+6y=12$ and $2y+3x=12$

$4x-6y=-12$

$m=\frac{4}{6}=\frac{2}{3}$

$3x+2y=12$

$m=\frac{-3}{2}$

$\boxed{\perp}$

3. Write the *standard form* of the line that passes through $(5,-1)$ and is $\perp$ to $3x-4y=12$.

$m=\frac{3}{4}$

$y+1=-\frac{4}{3}(x-5)$

$\left(y+1=-\frac{4}{3}x+\frac{20}{3}\right)3$

$3y+3=-4x+20$

$\boxed{4x+3y=17}$

$\begin{array}{r}21\\ 5\\ \hline 105\end{array}$

4. Write the *slope-intercept form* of the line through $(-21,3)$ and is $\perp$ to $-5y-3x=4x+23$

$$\begin{aligned} -5y-3x&=4x+23 \\ +3x\quad &\quad +3x \end{aligned}$$

$\frac{-5y}{-5}=\frac{7x}{-5}+\frac{23}{-5}$

$y=\frac{-7}{5}x-\frac{23}{5}$

$m=-\frac{7}{5}$

$y-3=\frac{5}{7}(x+21)$

$y-3=\frac{5}{7}x+15$

$+3 \qquad +3$

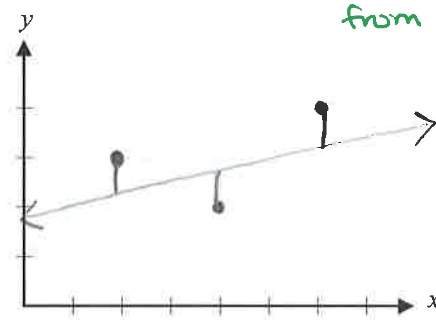
$\boxed{y=\frac{5}{7}x+18}$

Sections 2.5 – Using Linear Models

Objective: To write linear equations that model real-world data. To make predictions from linear models.

Given the table of values, graph the points on the x-, y-axes.

x	y
2	3
4	2
6	4



Congratulations! You just created a scatter plot.
Do the points connect to make a straight line? no
Draw a line that seems to fit the best.
Using a different color, **draw vertical segments** between the line and the points.

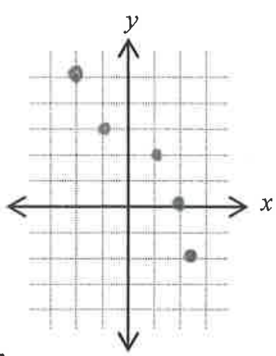
Least-Squares Line: $y =$ mx + b (a + bx) → statistics and $r =$ correlation coefficient

The correlation coefficient, represented by r, describes how closely the points cluster around the least-squares line. The r values will always be between −1 and 1, inclusive. show examples

Create a scatter plot of the data in the table below by hand. Describe the equation of the line and the correlation coefficient. Pick 2 points → point slope form → slope intercept form

1.

x	−2	−1	1	2	2.5
y	5	3	2	0	−2



* strength
direction
linear

$y =$ −1.33x + 2.27

$r =$ −0.96 (negative)

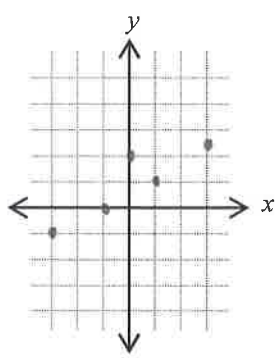
*strong negative linear *

negative correlation

⇒ a negative slope

2.

x	−3	−1	0	1	3
y	−1	0	2	1	2.5



$y =$ 0.58x + 0.9

$r =$ 0.90 (positive)

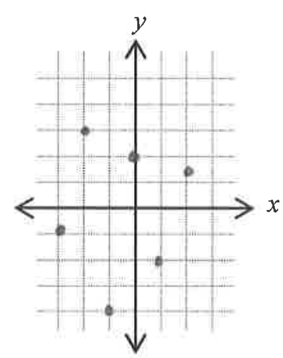
*strong positive linear *

positive correlation

⇒ a positive slope

3.

x	−3	−2	−1	0	1	2
y	−1	3	−4	2	−2	1.5



weak correlation

$r =$ 0.07 (close to zero)

no correlation

⇒ no possible slope

The closer r is to <u>-1</u> or <u>1</u>, the better the line fits the <u>points</u>. There is a <u>strong</u> correlation.

4. A baseball player has played baseball for several years. The following table shows his batting average for each year over a 10-year period.

x	8	9	10	11	12	13	14	15	16	17
Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
Batting Averag	0.250	0.258	0.262	0.280	0.272	0.278	0.285	0.292	0.316	0.320

a) Find the equation of the least-squares line (round to nearest thousandth). $y =$ <u>$0.007x + 0.190$</u>
Let $x = 0$ correspond to 1980.

$r = 0.957$

b) Describe the correlation. strong positive linear correlation.

Interpret: as he keeps playing, his batting average goes up.

c) Use the least-squares line to predict the baseball player's batting average in 1999.

$y = 0.007(19) + 0.190$

$= 0.323$ batting avg.

d) Use the least-squares line to predict *when* the player's batting average is 0.330.

$.330 = 0.007x + 0.190$

$.14 = 0.007x$

$20 = x \rightarrow \boxed{\text{year } 2000}$

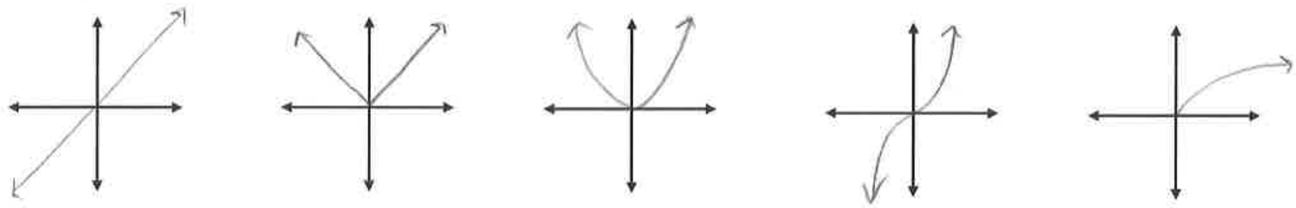
Sections 2.6 – Families of Functions

Objective: To analyze transformations of graphs.

Sketch each parent function. *You should know what these basic ones look like without making a table!*

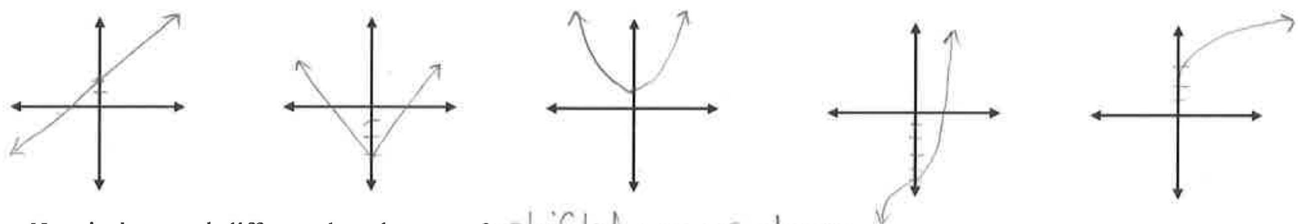
Parent Function

$f(x) = x$ $\quad$ $f(x) = |x|$ $\quad$ $f(x) = x^2$ $\quad$ $f(x) = x^3$ $\quad$ $f(x) = \sqrt{x}$



Graph each of the following and compare to the parent.

$g(x) = x + 2$ $\quad$ $g(x) = |x| - 3$ $\quad$ $g(x) = x^2 + 1$ $\quad$ $g(x) = x^3 - 5$ $\quad$ $g(x) = \sqrt{x} + 2.5$



How is the *graph* different than the parent? shifted up or down

How is the *equation* different than the parent? a value is added or subtracted

Transformation: vertical shift

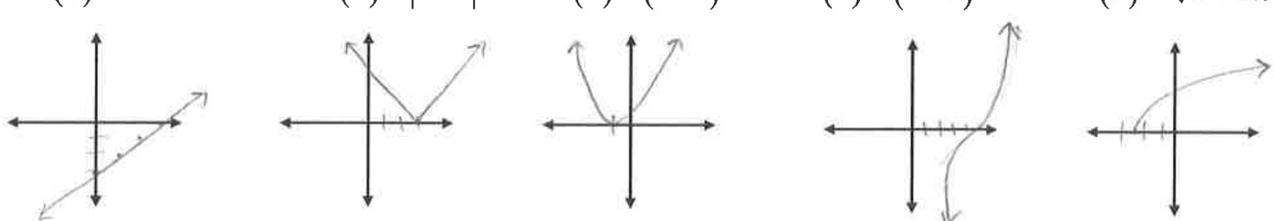
A vertical translation **or** vertical shift **moves the graph** up **or** down

The numbers in the equation are being added/subtracted to the "outside" **of the parent function.**

General Form: $y = f(x) + d$ → up **d units**
$f(x) - d$ → down **d units**

Graph each of the following and compare to the parent.

$h(x) = x - 3$ $\quad$ $h(x) = |x - 3|$ $\quad$ $h(x) = (x+1)^2$ $\quad$ $h(x) = (x-5)^3$ $\quad$ $h(x) = \sqrt{x + 2.5}$



How is the *graph* different than the parent? shifted right or left

How is the *equation* different than the parent? something added or subtracted inside

Transformation: horizontal

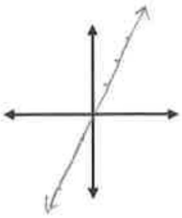
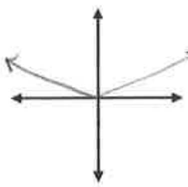
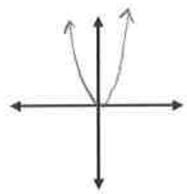
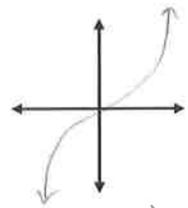
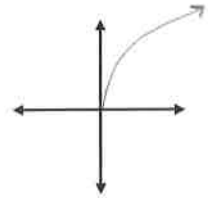
A horizontal translation **or** horizontal shift **moves the graph** left **or** right

The numbers in the equation are being added/subtracted to the "inside" **of the parent function.**

General Form: $y = f(x + c)$ → left **c units**
$y = f(x - c)$ → right **c units**

Graph each of the following and compare to the parent.

$g(x)=2x$ $\quad$ $g(x)=0.5|x|$ $\quad$ $g(x)=3x^2$ $\quad$ $g(x)=\frac{1}{4}x^3$ $\quad$ $g(x)=2.5\sqrt{x}$

How is the *graph* different than the parent? stretched or compressed

How is the *equation* different than the parent? parent function is multiplied by a value.

Transformation: vertical stretch/comp

A vertical stretch **or** comp. **changes the graph by** stretching **or** compressing **it.**

The numbers in the equation are being added/subtracted to the "outside" **of the parent function.**

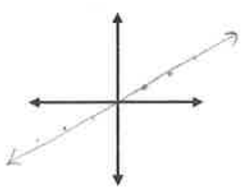
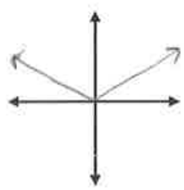
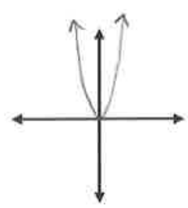
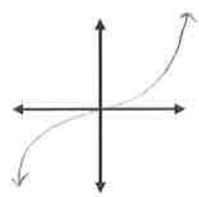
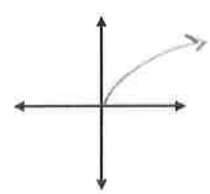
General Form: $y=a\cdot f(x)$

If $a<1$ **the graph is** compressed **by a factor of** a**.**

If $a>1$ **the graph is** stretched **by a factor of** a

Graph each of the following and compare to the parent.

$h(x)=\frac{1}{2}x$ $\quad$ $h(x)=|0.5x|$ $\quad$ $h(x)=(3x)^2$ $\quad$ $h(x)=\left(\frac{1}{4}x\right)^3$ $\quad$ $h(x)=\sqrt{2.5x}$

How is the *graph* different than the parent? stretched or compressed

How is the *equation* different than the parent? parent function is multiplied by a value "inside"

Transformation: horizontal stretch/comp

A horiz. stretch **or** comp. **changes the graph by** stretching **or** compressing **it.**

The numbers in the equation are being added/subtracted to the "inside" **of the parent function.**

General Form: $y=f(b\cdot x)$

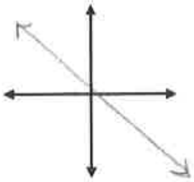
If $b<1$ **the graph is** Stretched **by a factor of** $\frac{1}{b}$**.**

If $b>1$ **the graph is** compressed **by a factor of** b **.**

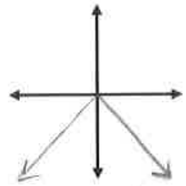
*** Vertical stretches are related to horizontal compressions, but they are not exactly the same thing, so we must rely on the equation and various transformations to find the specific transformation. As a rule of thumb, list the transformation (as vertical or horizontal) how it is given in the initial problem.***

Graph each of the following and compare to the parent.

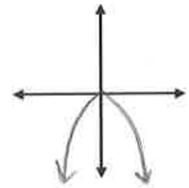
$g(x) = -x$



$g(x) = -|x|$



$g(x) = -x^2$



How is the *graph* different than the parent? flipped over the x-axis

How is the *equation* different than the parent? negative sign in front

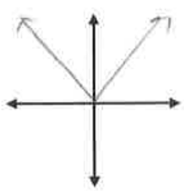
Transformation: Reflection across X axis

A negative **sign on the** "outside" **of the function** reflects **the graph over the** X axis.

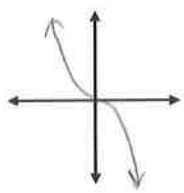
General Form: $y = -f(x)$

Graph each of the following and compare to the parent.

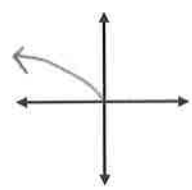
$h(x) = |-x|$



$g(x) = (-x)^3$



$h(x) = \sqrt{-x}$



How is the *graph* different than the parent? reflected over the y-axis

How is the *equation* different than the parent? negative sign "inside"

Transformation: Reflection across y-axis

A negative **sign on the** "inside" **of the function** reflects **the graph over the** y axis

General Form: $y = f(-x)$

General Transformation Equation: $y = a \cdot f(b(x+c)) + d$

a = vert stretch/comp

b = horiz stretch/comp

c = horiz shift

d = vert shift

Everything happening "outside" the function is a vertical **transformation.**

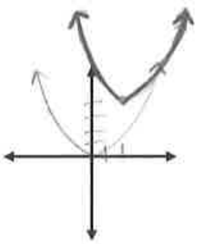
Everything happening "inside" the function is a horizontal **transformation.**

Compound Transformations:

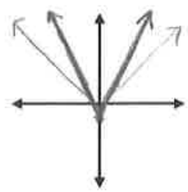
Describe the transformations in words and then graph the parent and the new graph on the same axes.

****When listing transformations, always do horizontal items first.***

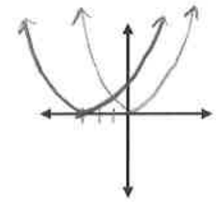
1. $g(x)=(x-2)^2+4$
vert. trans. up 4
Horiz. trans. right 2



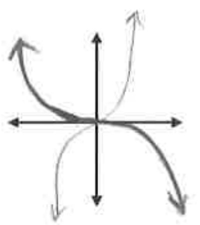
2. $h(x)=2|x|-1$
vert. stretch, a=2
vert. trans. down 1



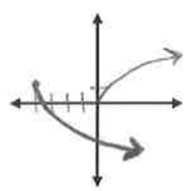
3. $j(x)=0.5(x+3)^2$
Vert comp, a = .5
horiz. trans. left 3



4. $p(x)=-\left(\frac{1}{4}x\right)^3$
reflection over x-axis
horiz. stretch by $\frac{1}{4}$



5. $w(x)=-2\sqrt{x+4}+1$
reflection over x-axis
vert. stretch, a=2
horiz. trans. left 4
vert. trans. up 1



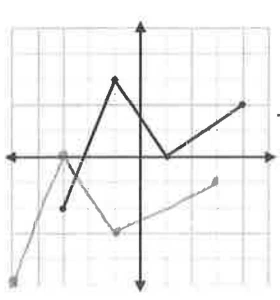
Additional problems:

Given the parent function $f(x)=x^3$, write the new function $g(x)$ based on the following transformations.

1. Horizontal shift left 5 units $g(x)=(x+5)^3$
2. Reflection across the x-axis and vertical shift down 2 units $g(x)=-x^3-2$
3. Vertical compression by a factor of 0.25 and a horizontal shift right 8 units $g(x)=.25(x-8)^3$

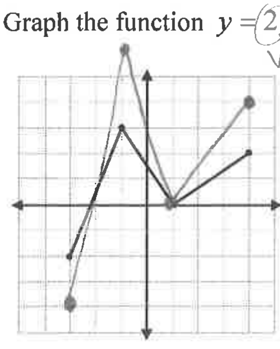
If $(2,-7)$ is a point on $y=f(x)$, then state the corresponding point on the new function. Hint: state the transformation(s).

4. $y=f(x)-3$ → V.T. down 3 → $(2,-10)$
5. $y=f(x+6)$ → H.T. left 6 → $(-4,-7)$
6. $y=5f(x)-3$ → V.S. factor of 5, V.T. down 3 → $(2,-35)$ → $\boxed{(2,-38)}$
7. Graph the function $y=f(x+2)-3$ V.T. down 3
H.T. left 2



$y=f(x)$

8. Graph the function $y=2f(x)$.
V.S. factor of 2

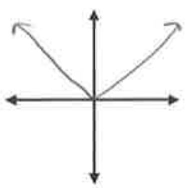
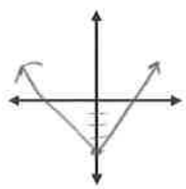
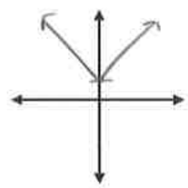
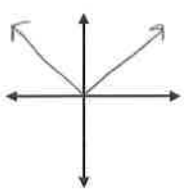
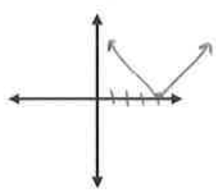
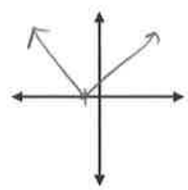


$y=f(x)$

Algebra 2 Name ______________________

Section 2.7 – Absolute Value Functions and Graphs

Sketch each function. Briefly describe the transformation from the parent function.

Parent Function			**General Form**
1. $f(x)=\lvert x\rvert$ 	$g(x)=\lvert x\rvert-4$ 	$h(x)=\lvert x\rvert+1$ 	$y=\lvert x\rvert+k$ → up k units $y=\lvert x\rvert-k$ → down k units
2. $f(x)=\lvert x\rvert$ 	$j(x)=\lvert x-4\rvert$ 	$k(x)=\lvert x+1\rvert$ 	$y=\lvert x+h\rvert$ → left h units $y=\lvert x-h\rvert$ → right h units

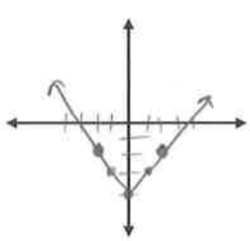
The simplest example of an absolute value function is $f(x) = |x|$. The graph of the absolute value of a linear function in two variable is V-shaped and symmetric about a vertical line called the axis of symmetry. Such a graph has either a single maximum point or single minimum point, called the vertex.

The transformation that were discussed in section 2.6, still apply here.

Make a table of values for each equation. Then graph the equation.

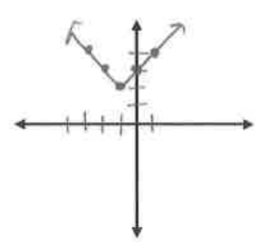
3. $y = |x| - 4$

x	y
-2	-2
-1	-3
0	-4
1	-3
2	-2



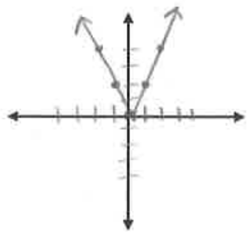
4. $y = |x + 1| + 2$

x	y
-3	4
-2	3
-1	2
0	3
1	4

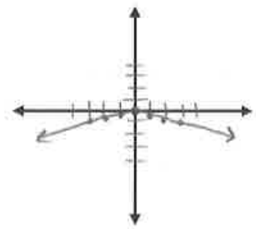


Graph each equation and then describe the transformation from the parent function $f(x) = |x|$.

5. $y = 2|x|$



6. $y = -\frac{1}{3}|x|$



General Form of the Absolute Value Function: $y = a|x-h| + k$

The stretch or compression factor is a, the vertex is located at (h, k), and the axis of symmetry is the line $x = h$.

Without graphing, identify the vertex, axis of symmetry, and transformations form the parent function $f(x) = |x|$.

7. $y = 4|x - 3| + 1$

Vertex: $(3, 1)$

a.o.s.: $x = 3$

Transformations:

V.S. factor of 4

H.T. right 3

V.T. up 1

8. $y = -\frac{3}{2}|x| - 1$

Vertex: $(0, -1)$

a.o.s.: $x = 0$

Transformations:

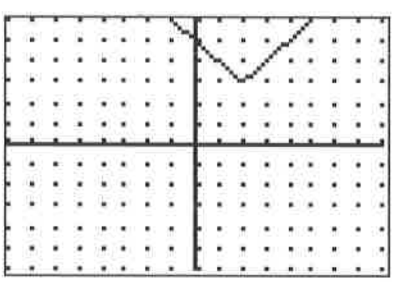
Reflection across x-axis

V.S. by factor of $\frac{3}{2}$

V.T. down 1

Write an absolute value equation for each graph. (think about the vertical and horizontal shifts, is it opening up or down, what's the slope of each side of the V)

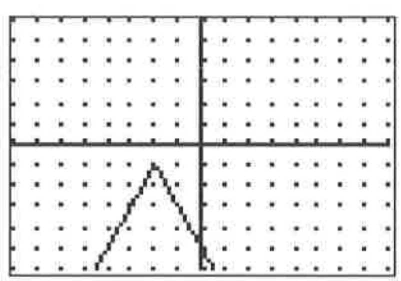
9.



$y = |x|$

$y = |x-2| + 3$

10.



$y = |x|$

$y = -2|x+2| - 1$

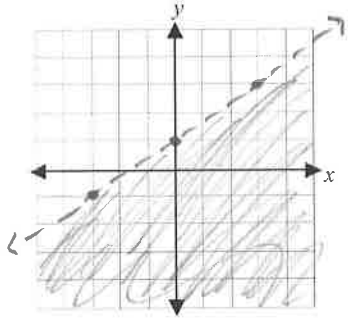
Sections 2.8 – Two-Variable Inequalities

Graphing linear inequalities in a Coordinate Plane

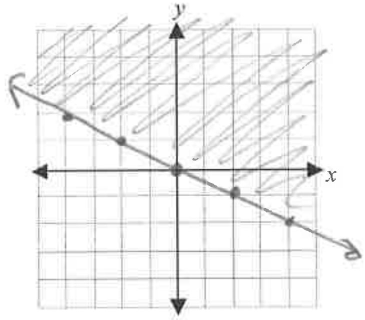
Summarize the steps:

1. Graph the boundary line.
2. Solid vs. dotted line.
   $\geq \leq$ $> <$
3. Shade the half-plane.
   Test a point not on the line such as (0 , 0).
   True $\Rightarrow$ shade that half
   False $\Rightarrow$ shade other half

1. $y < \frac{2}{3}x + 1$



2. $y \geq -\frac{1}{2}x$

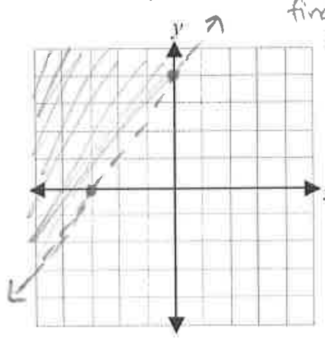


3. $4x - 3y < -12$

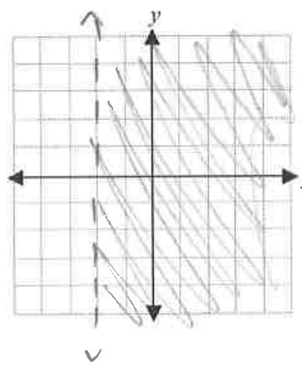
Plug in 0 to find intercepts or go to slope int.

$\frac{-3y}{-3} < \frac{-4x}{-3} \frac{-12}{-3}$

$y > \frac{4}{3}x + 4$



4. $x > -2$

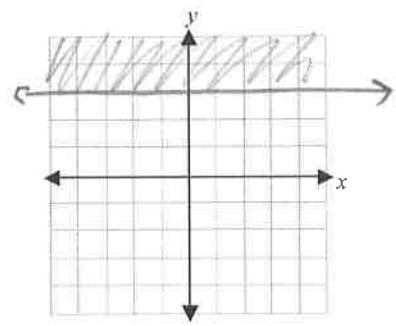


5. $3(4-2y) \leq -6$

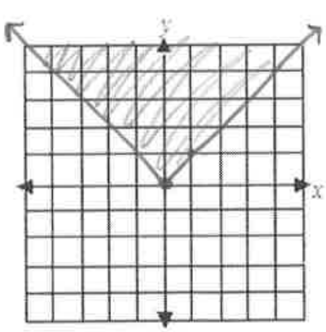
$12 - 6y \leq -6$

$\frac{-6y}{-6} \leq \frac{-18}{-6}$

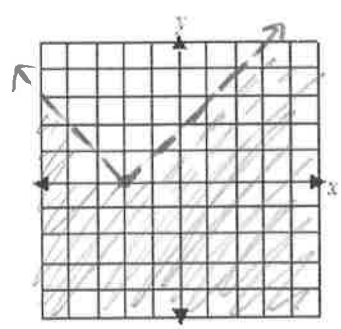
$y \geq 3$



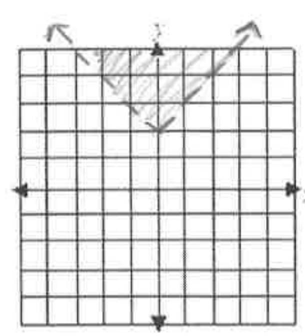
6. $y \geq |x|$



7. $y < |x+2|$



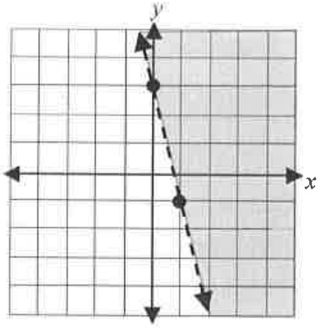
8. $y > |x| + 2$



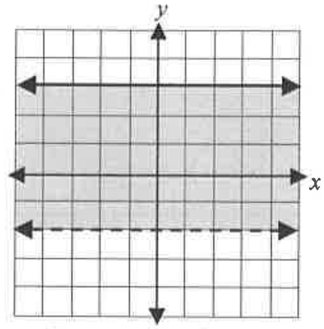
9. Write an inequality for each graph.

$m = -\frac{4}{1}$

$b = 4$

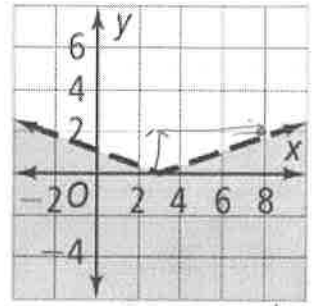


$y > -4x + 3$



$\begin{cases} y \le 3 \\ y > -2 \end{cases}$

$\boxed{-2 < y \le 3}$



$\frac{2}{5}$

$y \le \frac{2}{5}|x-3| + 0$

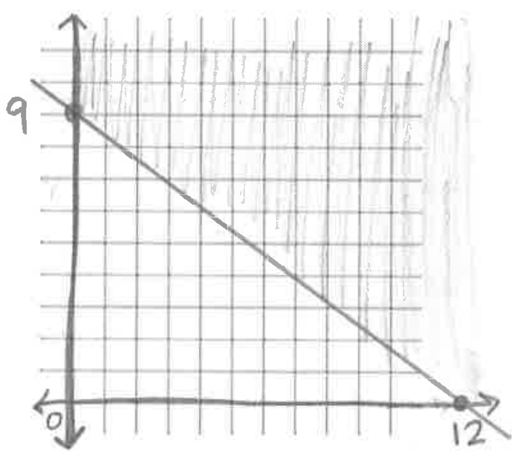
10. A salesperson sells two models of vacuum cleaners. One brand sells for $150 each and the other sells for $200 each. The salesperson has a weekly sales goal of at least $1800.

a. Write an inequality relating the revenue from the vacuum cleaners to the sales goal.

b. Graph the inequality.

$y \ge 0$ ($200 models)

$x \ge 0$ ($150 models)



$150x + 200y \ge 1800$

$150(0) + 200y \ge 1800$

$\frac{200y}{200} \ge \frac{1800}{200}$

$y \ge 9$

$(0,9)$

$150x + 200(0) \ge 1800$

$\frac{150x}{150} \ge \frac{1800}{150}$

$x \ge 12$

$(12,0)$

c. If the salesperson sold exactly six $200 models last week, how many $150 models did she have to sell to make her sales goal?

$150x + 200y \ge 1800$

$150x + 200(6) \ge 1800$

$150x + 1200 \ge 1800$

$\frac{150x}{150} \ge \frac{600}{150}$

$x \ge 4$

At least 4 $150 models.

Objective: To use dimensional analysis to convert units.

Dimensional Analysis

Real-world problems often involve units of measure and performing operations with these units is called dimensional analysis or unit conversion. The set-up of converting units is to write equivalences. Always include the units.

Example: Here are some examples of unit equivalences

1 foot = 12 inches 1 hour = 60 minutes 1 ton = 2,000 lbs

12 eggs = 1 dozen eggs 1 kilometer = 1000 meters 1 minute = 60 seconds

Converting Units 1 inch = 2.54 cm

To perform the conversion from one unit to another unit, you write the equivalences as a ratio. Since each of the equivalences is equal, the ratio is equal to 1.

For example: $\frac{1 \text{ foot}}{12 \text{ inches}} = \frac{12 \text{ inches}}{1 \text{ foot}} = 1$

Notice that you can write your equivalences as two different ratios that are equal. The purpose when converting is to cancel out the unwanted units

Example: Convert the following

1 week = 7 days

weeks → days → hours → mins → secs.

1. How many seconds in 5 weeks

$$\frac{5 \text{ weeks}}{1}\left(\frac{7 \text{ days}}{1 \text{ week}}\right)\left(\frac{24 \text{ hours}}{1 \text{ day}}\right)\left(\frac{60 \text{ min}}{1 \text{ hour}}\right)\left(\frac{60 \text{ sec}}{1 \text{ min}}\right) = 5 \cdot 7 \cdot 24 \cdot 60 \cdot 60$$

$\boxed{3{,}024{,}000 \text{ sec}}$

2. How many inches in one mile 1 mile = 5,280 ft

$$\frac{1 \text{ mile}}{1}\left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right)\left(\frac{12 \text{ inches}}{1 \text{ ft}}\right) = 5280 \cdot 12$$

$\boxed{63{,}360 \text{ in}}$

3. Convert 65 miles per hour to feet per hour.

$$\frac{65 \text{ miles}}{\text{hour}}\left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right) = 65 \cdot 5280 = \boxed{\frac{343{,}200 \text{ ft}}{1 \text{ hour}}}$$

4. Convert 70 miles per hour to feet per second

$$\frac{70 \text{ miles}}{1 \text{ hour}}\left(\frac{5280 \text{ ft}}{1 \text{ mile}}\right)\left(\frac{1 \text{ hour}}{60 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = \frac{70 \cdot 5280 \text{ ft}}{60 \cdot 60 \text{ sec}}$$

$\boxed{= 102.7 \frac{\text{ft}}{\text{sec}}}$

→

5. 100 m dash in 10 sec. MPH?

1 meter = 3.2 feet

$$\frac{100\,\cancel{m}}{10\,\cancel{sec}}\left(\frac{3.2\,\cancel{feet}}{1\,\cancel{meter}}\right)\left(\frac{60\,\cancel{sec}}{1\,\cancel{min}}\right)\left(\frac{60\,\cancel{min}}{1\,hour}\right)\left(\frac{1\,mile}{5280\,\cancel{ft}}\right) = \frac{100 \cdot 3.2 \cdot 60 \cdot 60}{10 \cdot 5280}$$

$$\boxed{= 21.8 \text{ mph}}$$