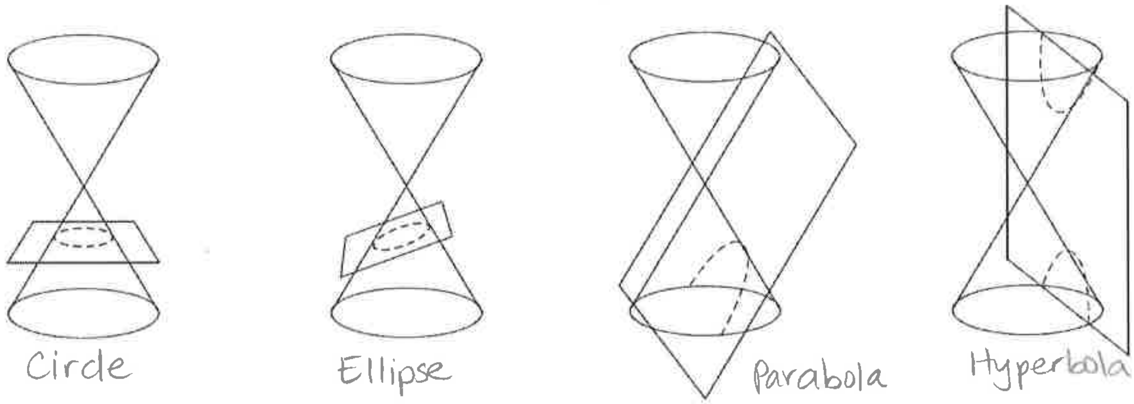
Chapter 10 – Conic Sections

The word *conic* derives from the word cone. *Conics*, an abbreviation for conic sections are curves that result from the intersection of a right circular cone and a plane. Identify the conics illustrated in the diagrams below.



Circle Ellipse Parabola Hyperbola

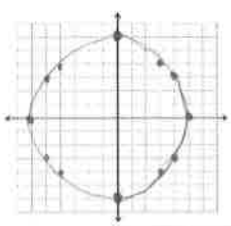
10.3 Notes Circles

Consider the equation: $x^2 + y^2 = 25$

Is it a line? Quadratic?
Exponential? Logarithmic? no!

Make a table and graph the equation.

x	y
0	± 5
3	± 4
4	± 3
5	0

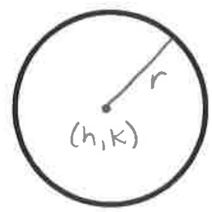


Definition of a Circle: The set of all points in a plane that are equidistant from a given point in the plane, called the center.

Equation in *standard form*: $(x-h)^2 + (y-k)^2 = r^2$

Where (h,k) is the **center** and r is the **radius**.

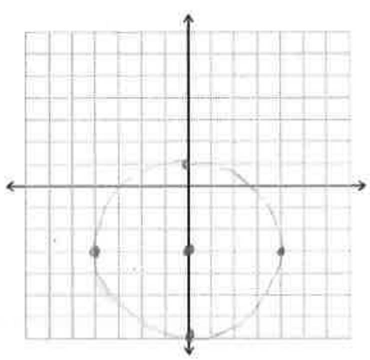
↑ opposite sign from equation



Example 1: Identify the center and radius of the circle. Then sketch the graph.

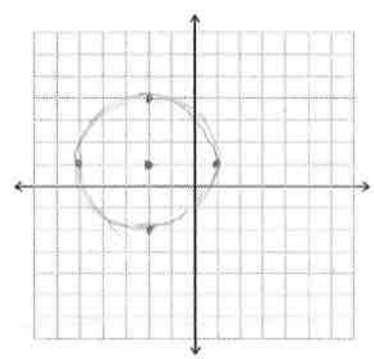
a.) $x^2 + (y+3)^2 = 16$

$C(0,-3)$ $r = 4$



b.) $(x+2)^2 + (y-1)^2 = 9$

$C(-2,1)$ $r = 3$



Example 2: Write the equation of the circle in standard form.

a.) $C(2, 0)$ and $r = 5$

$(x-2)^2 + y^2 = 25$

b.) $C(2, -4)$ and $r = 9$

$(x-2)^2 + (y+4)^2 = 81$

Review: **Completing the Square**

Perfect Trinomial Squares

$a^2 + 2ab + b^2$	$a^2 - 2ab + b^2$
$(a+b)(a+b)$	$(a-b)(a-b)$
$(a+b)^2$	$(a-b)^2$

Examples:

a.) $x^2 + 8x + \underline{4^2} = (x + 4)^2$

b.) $x^2 - 10x + \underline{5^2} = (x - 5)^2$

or $(-5)^2$

To Complete the Square:

1. Take the middle term, cut it in half and square it.
2. Add to both sides of the equation.
3. Change to factored form

We use the method of completing the square when we want to change the equation of a circle to Standard form.

Example 3: Write the standard equation of the circle. Identify the center and radius and then graph.

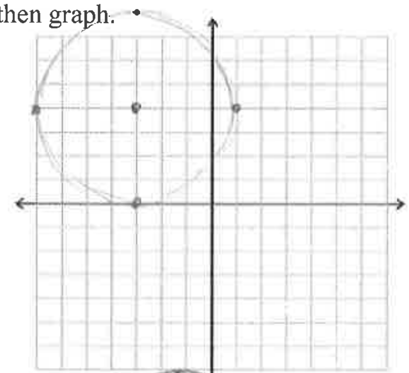
a.) $x^2 + y^2 + 6x - 8y + 9 = 0$

$x^2 + 6x + \_\_\_ + y^2 - 8y + \_\_\_ = -9$

$x^2 + 6x + \underline{3^2} + y^2 - 8y + \underline{4^2} = -9 + 3^2 + 4^2$

$(x+3)(x+3) + (y-4)(y-4) = 16$

$\boxed{(x+3)^2 + (y-4)^2 = 16}$ $\boxed{C(-3,4);\ r = 4}$

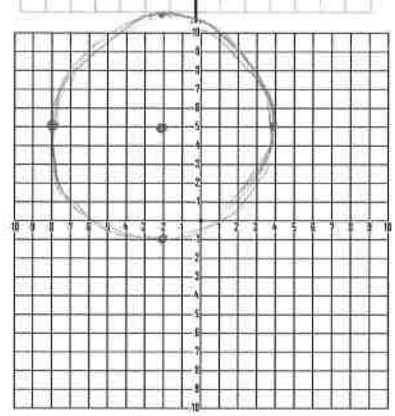


b.) b.) $x^2 + y^2 + 4x - 10y - 7 = 0$

$x^2 + 4x + \_\_\_ + y^2 - 10y + \_\_\_ = 7$

$x^2 + 4x + \underline{2^2} + y^2 - 10y + \underline{5^2} = 7 + 2^2 + 5^2$

$\boxed{(x+2)^2 + (y-5)^2 = 36}$ $\boxed{C(-2,5);\ r = 6}$



10.4 Notes Ellipses

Definition of an Ellipse: The set of all points *P* in a plane such that the sum of the distances from two fixed points is constant. The two fixed points are called the foci.

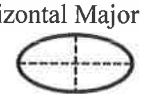
Properties of an Ellipse:

1.) Two axes of symmetry called the major axis (longer) and minor axis (shorter).

2.) The endpoints of the major axis are called the vertices, and the endpoints of the minor axis are called the co-vertices.

3.) The center of the ellipse is the intersection of the two axes of symmetry.

*Horizontal Major Axis

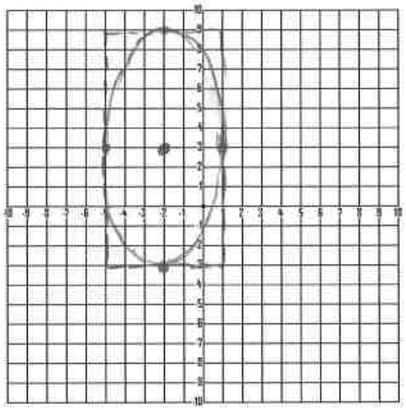


*Vertical Major Axis

Equation in *standard form:* $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ where (h,k) is the center, a (under x) is the horizontal stretch, and b (under y) is the vertical stretch.

Example 1: Identify the center, the horizontal stretch, the vertical stretch, and graph.

a.) $\frac{(x+2)^2}{9} + \frac{(y-3)^2}{36} = 1$

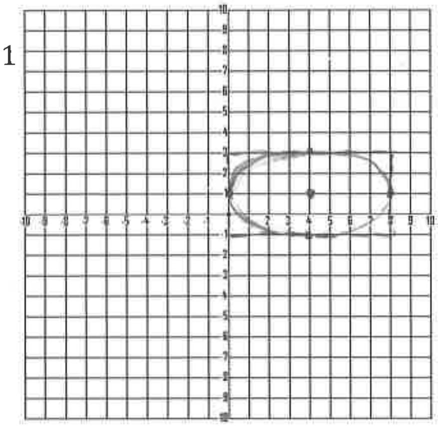


Center: (-2 , 3)

$\leftrightarrow$ = 3

$\updownarrow$ = 6

b.) $\frac{(x-4)^2}{16} + \frac{(y-1)^2}{4} = 1$

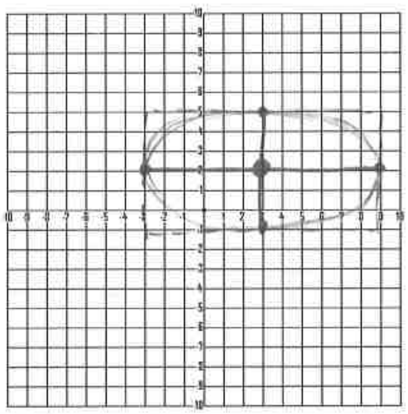


Center: (4 , 1)

$\leftrightarrow$ = 4

$\updownarrow$ = 2

Example 2: Write an equation for the ellipse with endpoints of the **major** axis at (-3, 2) and (9, 2) and endpoints of the **minor** axis at (3, -1) and (3, 5). HINT: Sketch a graph.



C(3,2)

$\leftrightarrow$ = 6

$\updownarrow$ = 3

$$\boxed{\frac{(x-3)^2}{36} + \frac{(y-2)^2}{9} = 1}$$

Example 3: Write the equation of the ellipse in standard form and then graph.

Reminder: **Factor out any** coefficients **attached to x^2 or y^2** BEFORE **completing the square!**

a.) $x^2 + 4y^2 + 4x - 24y + 24 = 0$

$(x^2 + 4x + \_\_) + 4(y^2 - 6y + \_\_) = -24$

$(x^2 + 4x + 2^2) + 4(y^2 - 6y + 3^2) = -24 + 2^2 + 4(3^2)$

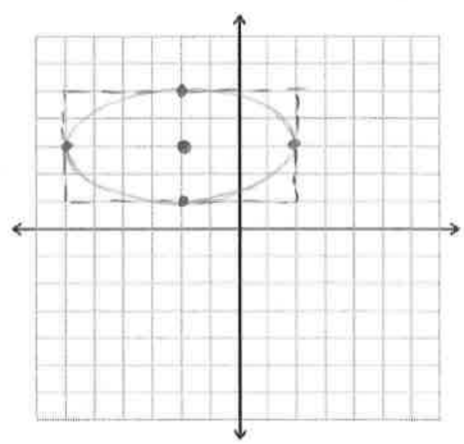
$\frac{(x+2)^2}{16} + \frac{4(y-3)^2}{\cancel{16}\ 4} = \frac{16}{16}$ ← needs to be 1

$$\boxed{\frac{(x+2)^2}{16} + \frac{(y-3)^2}{4} = 1}$$

C(-2,3)

$\leftrightarrow$ a = 4

$\updownarrow$ b = 2



b.) $4x^2 + 25y^2 - 24x + 200y + 336 = 0$

$4(x^2 - 6x + \_\_) + 25(y^2 + 8y + \_\_) = -336$

$4(x^2 - 6x + 3^2) + 25(y^2 + 8y + 4^2) = -336 + 4(3^2) + 25(4^2)$

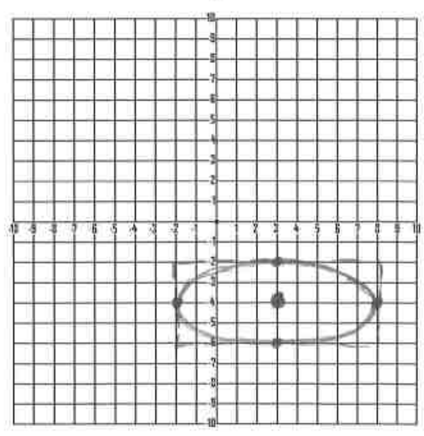
$\frac{\cancel{4}(x-3)^2}{\cancel{100}\ 25} + \frac{\cancel{25}(y+4)^2}{\cancel{100}\ 4} = \frac{100}{100}$ ← needs to be 1

$$\boxed{\frac{(x-3)^2}{25} + \frac{(y+4)^2}{4} = 1}$$

C(3,-4)

$\leftrightarrow$ a = 5

$\updownarrow$ b = 2



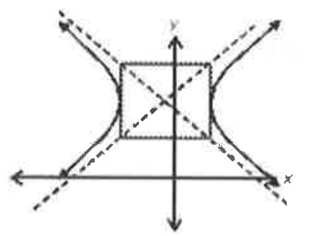
10.5 Notes Hyperbolas

Definition of an Hyperbola: The set of all points P in a plane such that the absolute value of the difference between the distance from two fixed points is constant. The two fixed points are called the foci.

Standard Form

Opens Left/Right (x comes first)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



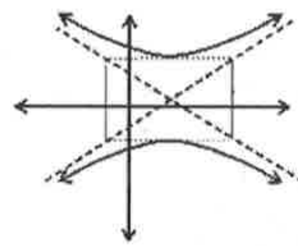
Center: (h, k)

Horizontal Stretch: a
↳ under x

Vertical Stretch: b
↳ under y

Opens Up/Down (y comes first)

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$



Example 1: Identify the center, the horizontal stretch, the vertical stretch, and then graph.

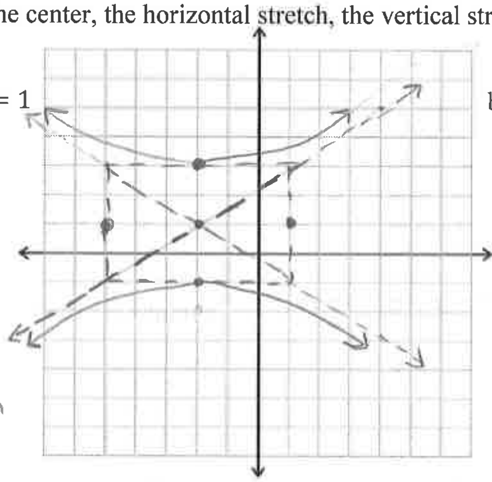
a.) $\frac{(y-1)^2}{4} - \frac{(x+2)^2}{9} = 1$

Center: (-2 , 1)

↔ = 3

↕ = 2

opens up/down



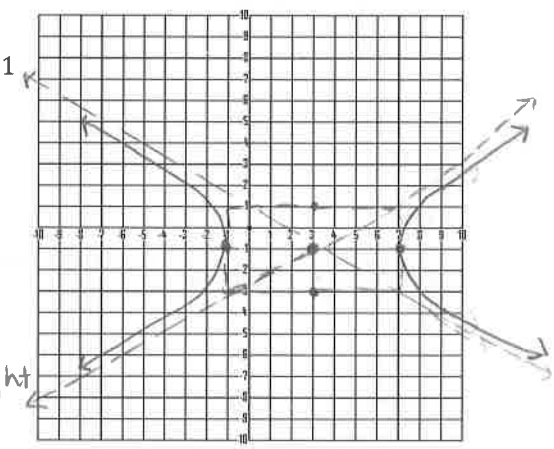
b.) $\frac{(x-3)^2}{16} - \frac{(y+1)^2}{4} = 1$

Center: (3 , -1)

↔ = 4

↕ = 2

opens left/right



Example 2: Write the equation in standard form and then graph.

a.) $x^2 - 4y^2 + 8x + 16y - 4 = 0$

$(x^2 + 8x + \_\_) - 4(y^2 - 4y + \_\_) = 4$

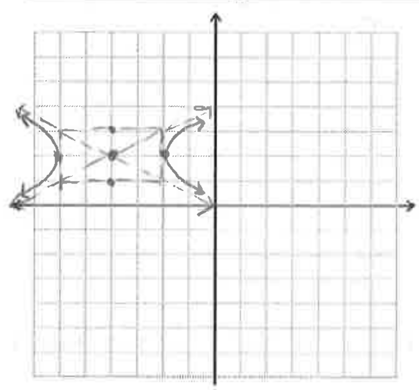
$(x^2 + 8x + 4^2) - 4(y^2 - 4y + 2^2) = 4 + 4^2 - 4(2)^2$

$\frac{(x+4)^2}{4} - \frac{4(y-2)^2}{4} = \frac{4}{4}$ ← needs to be 1

$\boxed{\frac{(x+4)^2}{4} - \frac{(y-2)^2}{1} = 1}$

C(-4, 2)
↔ a = 2
↕ b = 1

opens left/right



b.) $9y^2 - 4x^2 - 8x - 36y - 4 = 0$

$9(y^2 - 4y + \_\_) - 4(x^2 + 2x + \_\_) = 4$

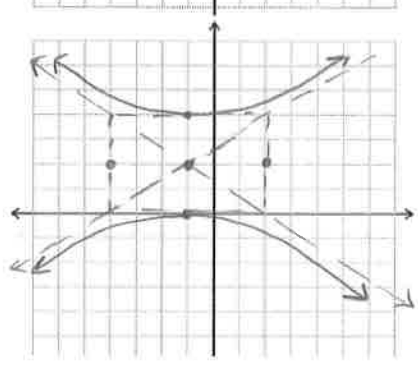
$9(y^2 - 4y + 2^2) - 4(x^2 + 2x + 1^2) = 4 + 9(2)^2 - 4(1)^2$

$\frac{\cancel{9}(y-2)^2}{\cancel{36}\,4} - \frac{\cancel{4}(x+1)^2}{\cancel{36}\,9} = \frac{36}{36}$ ← needs to be 1

$\boxed{\frac{(y-2)^2}{4} - \frac{(x+1)^2}{9} = 1}$

C(-1, 2)
↔ a = 3
↕ b = 2

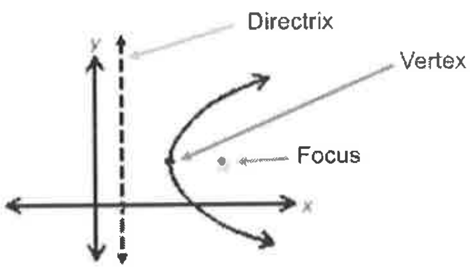
opens up/down

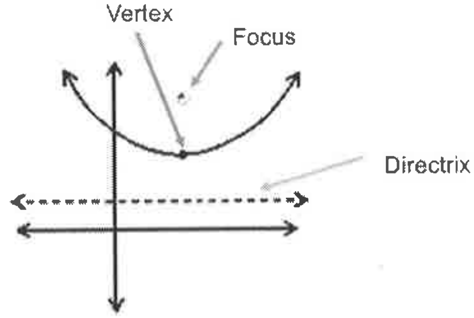


10.2 Notes Parabolas

Definition of a Parabola: A parabola is the set of all points in a plane that are the same distance from a given point called the focus, and a given line called the directrix. Parabolas can open up, down, left, or right.

The parabola opens around the focus.





The distance from vertex to the focus is the same as the distance from the vertex to the directrix.

Standard Form Equation

Note: Standard form change depending on whether the parabola opens up, down, left, or right.

Up OR Down	$y-k = \pm \frac{1}{4a}(x-h)^2$	*Y comes first *If $\frac{1}{4a}$ is positive, opens up *If $\frac{1}{4a}$ is negative, opens down
Right OR Left	$x-h = \pm \frac{1}{4a}(y-k)^2$	*X comes first *If $\frac{1}{4a}$ is positive, opens right *If $\frac{1}{4a}$ is negative, opens down

- The vertex is the point (h, k)
- *a* is the distance from the *vertex* to the *focus*, as well as the distance from the *vertex* to the *directrix*.
- $\frac{1}{4}$ is a constant.

Example 1: Identify the vertex, focus, and the directrix (equation) of the parabola with the given equation. Then sketch the graph.

a.) $\boxed{\text{up}}$ $y = \frac{1}{16}x^2$

$\frac{1}{16} = \frac{1}{4a}$

$\frac{16}{4} = \frac{4a}{4}$

$a = 4$

Vertex: $(0, 0)$

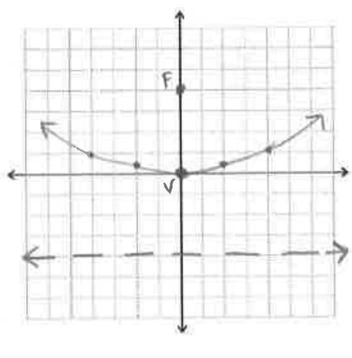
Focus: $(0, 4)$

Directrix: $y = -4$

Plug in an x to find another point

$y = \frac{1}{16}(2)^2$ $\quad$ $y = \frac{1}{16}(4)^2$

$y = \frac{1}{4}$ $\quad$ $y = 1$



b.) $\boxed{\text{left}}$ $x + 2 = -\frac{1}{4}(y+1)^2$

$\frac{1}{4} = \frac{1}{4a}$

$\frac{4}{4} = \frac{4a}{4}$

$a = 1$

Vertex: $(-2, -1)$

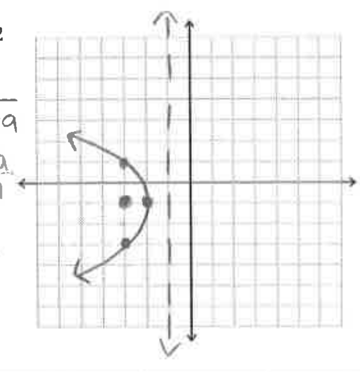
Focus: $(-3, -1)$

Directrix: $x = -1$

$x + 2 = -\frac{1}{4}(1+1)^2$

$x + 2 = -\frac{1}{4}(4)$

$x = -3$



Example 2: Write an equation of a parabola with vertex at the origin and the given focus. HINT: Sketch a graph!

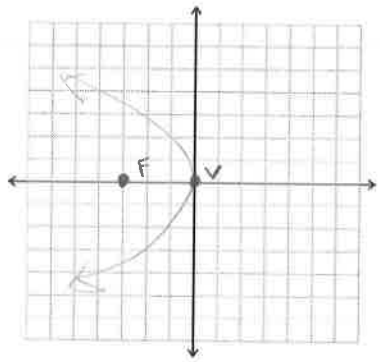
Left

a.) Focus at (-3,0)

$V(0,0)$

$x+0 = -\frac{1}{4(3)}(y+0)^2$

$\boxed{x = -\frac{1}{12}y^2}$



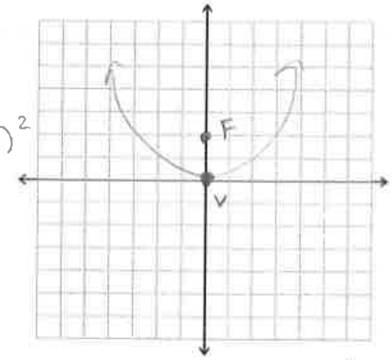
up

b.) Focus at (0, 2)

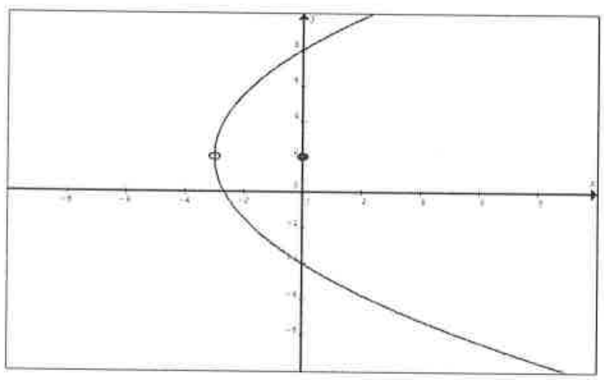
$V(0,0)$

$y+0 = +\frac{1}{4(2)}(x-0)^2$

$\boxed{y = \frac{1}{8}x^2}$



Example 3: Write the equation in standard form for the graph provided below.



Vertex $(-3,2)$

Focus $(0,2)$

opens right

$x+3 = \frac{1}{4(3)}(y-2)^2$

$\boxed{x+3 = \frac{1}{12}(y-2)^2}$

Completing the Square:

1.) Move the squared variable with its linear term to one side of the equation.

2.) Move the constant term and the other variable to the other side of the equation.

3.) Complete the square for the squared term. You only need to complete the square ONCE !

4.) Divide both sides by the number attached to the Linear term.

Example 4: Write the equation of the parabola in standard form and then graph. Be sure to include all important parts on your graph!

a.) $y^2 - 12x - 4y - 32 = 0$

$y^2 - 4y = 12x + 32$

$y^2 - 4y + \_\_\_ = 12x + 32$

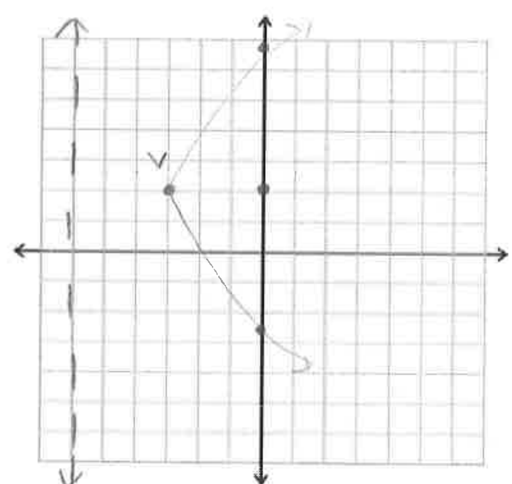
$y^2 - 4y + 2^2 = 12x + 32 + 2^2$

$\frac{(y-2)^2}{12} = \frac{12x}{12} + \frac{36}{12}$

$\frac{(y-2)^2}{12} = x + 3$

$\boxed{x+3 = \frac{1}{12}(y-2)^2}$

opens right



Solve for a to find focus:

$\frac{1}{12} = \frac{1}{4a}$

$12 = 4a$

$a = 3$

Vertex: $(-3,2)$

Focus: $(0,2)$

Directrix: $x = -6$

$x+3 = \frac{1}{12}(0-2)^2$

$x+3 = \frac{1}{12}(4)$

$x+3 = \frac{1}{3}$

$x = -2\frac{2}{3}$