Date: ______________ Notes 11.1 Permutations + Combinations

Find the number of possible outcomes.

The cafeteria has decided to offer a *lunch tray* that has 1 entrée and 1 fruit.

Students can choose from the following:
entrée: lasagna, hamburger or chicken
fruit: orange or apple

Draw a **tree diagram** to illustrate the possible outcomes.

lasagna < orange, apple
hamburger < orange, apple
chicken < orange, apple

How many different lunch trays are possible? 6

Do you see an easy way to calculate this number? 3 X 2

Fundamental Counting Principle (FCP)

If there are m ways that one event can occur, and n ways that another event can occur, then there are m x n ways that both events can occur.

The **FCP** applies to more than two events occurring.
Example: Suppose you must choose a password for your computer. You cannot use the letters A and Z or the digits 0 and 9. How many possible passwords are there if the password is 3 letters followed by 3 digits and

1. repetition of letters and numbers is allowed?

minus A and Z → minus 0,9

$\underset{1^{st}\text{ ltr}}{24} \cdot \underset{2^{nd}\text{ ltr}}{24} \cdot \underset{3^{rd}\text{ ltr}}{24} \cdot \underset{1^{st}\,\#}{8} \cdot \underset{2^{nd}\,\#}{8} \cdot \underset{3^{rd}\,\#}{8} = \boxed{7{,}077{,}888}$

2. no repetition is allowed?

$24 \cdot 23 \cdot 22 \cdot 8 \cdot 7 \cdot 6 = \boxed{4{,}080{,}384}$

*no repetition = # of choices decreases with each blank

Factorial notation: Be able to do this both *with* and *without your calculator*.

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{120}$

$\frac{9!}{6!} = \frac{9 \cdot 8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 9 \cdot 8 \cdot 7 = \boxed{504}$

$\frac{25!}{20! \times 5!} = \frac{\overset{5}{\cancel{25}} \cdot \cancel{24} \cdot 23 \cdot 22 \cdot 21 \cdot \cancel{20!}}{\cancel{20!} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 5 \cdot 23 \cdot 22 \cdot 21 = \boxed{53{,}130}$

$\frac{3!}{(4-4)!} = \frac{3!}{0!} = \frac{3!}{1} = 3 \cdot 2 \cdot 1 = \boxed{6}$

By definition, $0! = 1$

How many different ways can you arrange the letters in the word **ANT**? Show all the possibilities.

ANT NAT TNA
ATN NTA TAN = $\boxed{6}$

Use the FCP to find the number of arrangements: $\underset{3\text{ choices}}{3} \cdot \underset{2\text{ choices}}{2} \cdot \underset{1\text{ choice}}{1} = 3! = 6$

Use the FCP to determine the number of arrangements of the word: **MATH**: $4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$

Permutations of n Objects

Permutation is an arrangement of objects where order matters.

The **number of permutations of n distinct objects** is $n!$.

How many different arrangements/permutations are there of the word **MONEY**? $5! = 120$

Use the FCP to determine the number of permutations if you can only use

3 of the letters: $5 \cdot 4 \cdot 3 = 60$

4 of the letters: $5 \cdot 4 \cdot 3 \cdot 2 = 120$

How can we get these answers using factorial notation?

5 letters taken 3 at a time: $5 \cdot 4 \cdot 3 = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{5!}{2!} = \frac{5!}{(5-3)!}$

Permutations of n Objects Taken r at a Time

The **number of permutations of n objects taken r at a time** is ${}_nP_r = \frac{n!}{(n-r)!}$

Find ${}_6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = \frac{6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 6 \cdot 5 = \boxed{30}$

${}_7P_5 = \frac{7!}{(7-5)!} = \frac{7!}{2!} = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = \boxed{2{,}520}$

${}_3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 3! = \boxed{6}$

On a baseball team, 9 players are designated as the starting line-up. Before a game, the manager announces the *order* in which the 9 players will bat. How many different orders are there?

${}_9P_9 = \frac{9!}{(9-9)!} = 9! = \boxed{362{,}880}$

If there are 15 players on the baseball roster, how many different starting line-ups are there?

${}_{15}P_9 = \frac{15!}{(15-9)!} = \frac{15!}{6!} = 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = \boxed{1{,}816{,}214{,}400}$

How many *different* arrangements are there for the word **MOM**? Show all arrangements.
How is this different than the example with the word ANT? * identical letters

MOM
MMO
OMM

$\boxed{3}$

Use factorial notation to find the solution.

3 letters → 3! ways to arrange
2 M's → 2! ways to arrange

$\frac{3!}{2!} = \boxed{3}$

How about the word **DEED**? Show all arrangements. Use factorial notation to find the solution.

DEED EDED
DEDE EEDD
DDEE EDDE

4 letters → 4! arrangements
2 D's → 2!
2 E's → 2!

$\frac{4!}{2!\,2!} = \frac{\overset{2}{\cancel{4}} \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot 1 \cdot \cancel{2} \cdot \cancel{1}} = \boxed{6}$

How many different arrangements for the word **BANANA**?

6 letters, 3 A's, 2N's

$\frac{6!}{3!\,2!} = \frac{6 \cdot 5 \cdot \overset{2}{\cancel{4}} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot \cancel{2} \cdot 1} = \boxed{60}$

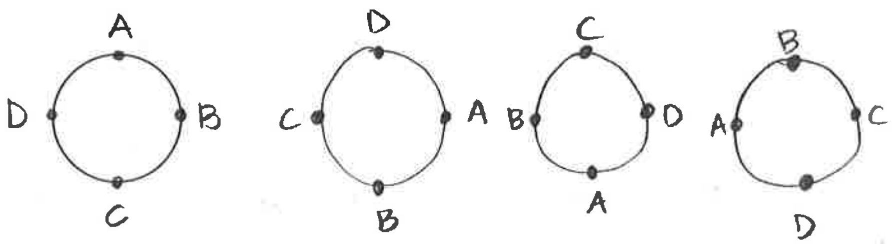
Permutations With Identical Objects

If there are n objects, then there are $\underline{n!}$ permutations, but if some of the **objects are identical,** r_1 of one kind of identical object, r_2 of a second kind of identical object, … then the number of distinct permutations is $\dfrac{n!}{r_1!\, r_2! \ldots}$

Suppose you plant 11 tulips in a row. If 4 of the tulips are red, 5 are yellow, and 2 are purple, how many different arrangements are there (assume that each type of tulip is identical)?

$$\frac{11!}{4!\,5!\,2!} = \boxed{6930}$$

Suppose 4 people are asked to sit in a *circle*. How many different ways can the people be arranged?



→ these are the same, not considered a different arrangement.

$$\frac{4!}{4} = 3! = \boxed{6}$$

Circular Permutations

If there are n distinct objects, then there are $\underline{n!}$ different permutations, but if we arrange them in a **circle**, then there are $\underline{(n-1)!}$ circular permutations of the n objects.

How many ways can 12 different spices be arranged around a *circular* spice rack?

$$(12-1)! = 11! = \boxed{39{,}916{,}800}$$

Compare the following:

order matters

1. I have 5 CDs and decide to listen to just 2 of them. How many different ways can I listen to them?

$${}_5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = 5 \cdot 4 = \boxed{20}$$

2. I decide to buy some CDs. I have narrowed my selection down to 5 CDs, but only have cash for 2 CDs. If we name the CDs: A, B, C, D, and E… list the different possibilities. Does *order* matter? No, AB + BA are the same purchase

AB BC CD DE
AC BD CE
AD BE
AE

$\boxed{\text{10 possibilities}}$

Can we use factorial notation to find the solution? ${}_5P_2$ → need to divide out the duplicates → 2!

$$\frac{5!}{2!\,(5-2)!} = \frac{5!}{2!\,3!} = \boxed{10}$$

Combinations of n Objects Taken r at a Time

Combination is an arrangement where order does not matter.

The **number of combinations of n objects taken r at a time** is ${}_nC_r = \frac{n!}{r!(n-r)!}$

Another way to write this is: $\binom{n}{r}$

Find ${}_9C_7 = \frac{9!}{7!(9-7)!} = \frac{9!}{7!\,2!} = \frac{9\cdot 8\cdot \cancel{7!}}{\cancel{7!}\,2!} = \boxed{36}$

Which is larger ${}_9P_7$ or ${}_9C_7$? ${}_9P_7 = \frac{9!}{2!} = \boxed{181{,}440}$

Why? Because order matters, there are more possibilities for different arrangements

Permutations vs. Combinations

1. How many different ways are there to give a 1st place, 2nd place, and 3rd place prizes to a group of 8 participants? [Hint: does order matter?] Yes → perm.

   ${}_8P_3 = \frac{8!}{5!} = 8\cdot 7\cdot 6 = \boxed{336}$

2. How many different ways are there to give 3 honorable mention awards to a group of 8 participants? [Hint: does order matter?] No → comb.

   ${}_8C_3 = \frac{8!}{5!\,3!} = \frac{8\cdot 7\cdot \cancel{6}}{\cancel{3}\cdot\cancel{2}} = \boxed{56}$

3. How many ways can I purchase 2 CDs, 3 DVDs, and 1 book from 7 CDs, 5 DVDs, and 3 books? [[Hint: does order matter?] Find the number of ways for each type, then use the FCP.

   $\boxed{\text{no}}$

   CDs: ${}_7C_2 = \frac{7!}{2!\,5!} = \boxed{21}$ DVDs: ${}_5C_3 = \frac{5!}{3!\,2!} = \boxed{10}$ Books: ${}_3C_1 = \frac{3!}{1!\,2!} = \boxed{3}$

   \# of ways purchased all together (use FCP): $\underset{\text{CD's}}{21} \cdot \underset{\text{DVD's}}{10} \cdot \underset{\text{Books}}{3} = \boxed{630}$

Date ______________ Notes 11.2: Probability

Probability is the likelihood or chance that an event will occur

Theoretical probability:

If an event, E, is random, then the probability of E is: $P(E) = \dfrac{\text{\# of outcomes of event } E}{\text{\# of outcomes in sample space}}$

Sample Space is the set of all possible outcomes.

Example: If you roll a six-sided die, find the following.

$P(3) = \frac{1}{6}$ $P(\text{even \#}) = \frac{3}{6} = \frac{1}{2}$ $P(\# > 6) = \frac{0}{6} = 0$ $P(\# < 7) = \frac{6}{6} = 1$ $P(\#\text{ is not } 3) = \frac{5}{6}$

$(1 - \frac{1}{6})$

Conclusions: If an event, E, is impossible, then $P(E) = \underline{0}$. (0%)

If an event, E, is certain (must happen), then $P(E) = \underline{1}$. (100%)

In general, $\underline{0} \le P(E) \le \underline{1}$

$P(\text{not } E) = P(E^c) = \underline{1} - \underline{P(E)}$ E^c = complement of E ("not E")

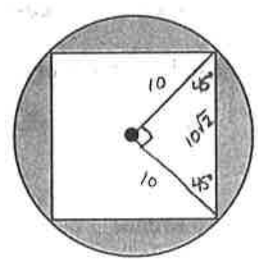
Example: Given a bag containing 8 blue marbles, 5 red marbles, and 2 orange marbles, find

$P(\text{orange}) = \frac{2}{15}$ $P(\text{yellow}) = 0$ $P(\text{blue, red or orange}) = 1$ $P(\text{not orange}) = 1 - \frac{2}{15}$

$= \frac{13}{15}$

Probability and Geometry

A square is inscribed in a circle of radius 10. Find the probability that a dart thrown at the circle will land into the shaded region. (nearest 10th of a percent).



$A_O = \pi(10)^2$

$= 100\pi$

$P(\text{shaded}) = \dfrac{\text{Area of shaded}}{\text{Area of total}}$

$A_\square = (10\sqrt{2})^2 = 200$

$= \dfrac{100\pi - 200}{100\pi} \approx \boxed{36.3\%}$

$A_{\text{shaded}} = A_O - A_\square$

Odds in favor of an event are defined as the number of ways the event can happen, *divided by* the number of ways it can fail to happen.

If the **odds in favor** of an event, E, are a to b, then the **probability** of the event is $P(E) = \dfrac{a}{a+b}$

Example: If the **odds** of the Giants beating the Dodgers is 2 to 1, find the **probability** that the Giants will beat the Dodgers. $\frac{2}{2+1} = \frac{2}{3}$

Find the **probability** that the Dodgers will beat the Giants. $1 - 2/3 = 1/3$

Combinations and Probability

A survey of 30 students found that 19 people favored the dance policy and 11 opposed it. Find the probability that in a random sample of 8 students from the survey, exactly 3 favor it and 5 oppose it. [Break it up into 3 steps.]

1. How many ways can 8 students be picked from the 30 in the survey? Does order matter? no

$$_{30}C_8 = \frac{30!}{8!\,22!} = \frac{\overset{5}{\cancel{30}}\cdot 29\cdot \overset{4}{\cancel{28}}\cdot 27\cdot 26\cdot \overset{5}{\cancel{25}}\cdot \cancel{24}\cdot 23}{\cancel{8}\cdot \cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = \boxed{5,852,925}$$

2. Two events: for and against

How many ways can you choose 3 from the 19 in favor?

$$_{19}C_3 = \frac{19!}{3!\,16!} = \frac{19\cdot \overset{3}{\cancel{18}}\cdot 17}{\cancel{3}\cdot \cancel{2}} = \boxed{969}$$

How many ways can you choose 5 from the 11 against?

$$_{11}C_5 = \frac{11!}{5!\,6!} = \frac{11\cdot \overset{2}{\cancel{10}}\cdot \overset{3}{\cancel{9}}\cdot \cancel{8}\cdot 7}{\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1} = \boxed{462}$$

3. Use the FCP to find the # of ways to choose the 3 students in favor and the 5 against.

$$969 \times 462 = \boxed{447,678}$$

Find the probability that exactly 3 students are in favor and 5 oppose.

$$P(3\text{ for}, 5\text{ oppose}) = \frac{447,678}{5,852,925} = 0.76 = \boxed{7.6\%}$$

Try this…

A bag of candy contains 6 Milky Way and 5 Snickers. (Assume the candy bars are numbered – each one is distinguishable). If you can choose 5 pieces of candy, find the probability of picking 2 Milky Way and 3 Snickers.

1. Find the number of ways that 5 pieces of candy can be picked from 11. Does order matter? no

$$_{11}C_5 = \frac{11!}{6!\,5!} = \boxed{462}$$

2. Two events:

Find the number of ways you can choose 2 Milky Way from 6. $_6C_2 = \frac{6!}{2!\,4!} = \frac{\overset{3}{\cancel{6}}\cdot 5}{\cancel{2}\cdot 1} = \boxed{15}$

Find the number of ways you can choose 3 Snickers from 5. $_5C_3 = \frac{5!}{3!\,2!} = \frac{5\cdot \overset{2}{\cancel{4}}}{\cancel{2}\cdot 1} = \boxed{10}$

3. Use the FCP to find the number of ways to choose the 2 Milky Way and 3 Snickers. $15 \times 10 = \boxed{150}$

Find the probability that you will pick exactly 2 Milky Way and 3 Snickers. $\frac{150}{462} = .325 = \boxed{32.5\%}$

Date ______________ Notes 11.3 Probability of Multiple Events

When do you add to find the probability?

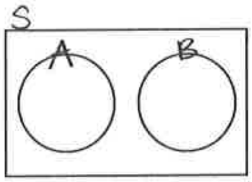
Suppose you roll a regular six-sided die. List the outcomes:

Find $P(2) = \frac{1}{6}$ $P(4) = \frac{1}{6}$ $P(\text{even } \#) = \frac{3}{6} = \frac{1}{2}$

$P(2 \text{ or } 4) = \frac{2}{6} = \frac{1}{3}$ $P(2 \text{ or even } \#) = \frac{3}{6} = \frac{1}{2}$

$\left[P(2) + P(4) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}\right]$

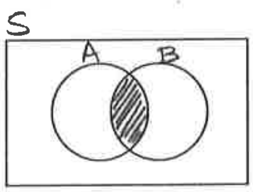
$P(2 \text{ or } 4)$ is an example of two events that are mutually exclusive. That means that the events from the same sample space have no intersection; they have nothing in common. (no overlap)



If A and B are **mutually exclusive** events, then $P(A \text{ or } B) = P(A) + P(B)$

ex: $P(2 \text{ or } 4) = P(2) + P(4)$

$P(2 \text{ or even } \#)$ is an example of two events that are inclusive. That means that the events from the same sample space do intersect; they have something in common. (overlap)



If A and B are **inclusive** events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

ex: $P(2 \text{ or even}) = P(2) + P(\text{even}) - P(2 + \text{even})$ ↑ the overlap

A survey was done of 250 sophomores and 250 juniors. They were asked if they liked the dress code, disliked it, or had no opinion. The results are given in the table below.

	Soph.	Juniors	Total
Liked	65	180	245
Disliked	157	54	211
No opinion	28	16	44
Total	250	250	500

add

What's the probability that a student selected at random likes or has no opinion on the dress code? Are the events *mutually exclusive* or *inclusive*?

$P(\text{Likes } or \text{ No opinion}) = P(L) + P(No)$

$= \frac{245}{500} + \frac{44}{500} = \boxed{\frac{289}{500}}$

add

What's the prob. that a student selected at random is a Junior or dislikes the dress code? *Mutually exclusive* or *inclusive*?

↑ the events can happen at the same time

$P(\text{Junior } or \text{ Dislikes}) = P(J) + P(DL) - P(J + DL)$

$= \frac{250}{500} + \frac{211}{500} - \frac{54}{500} = \boxed{\frac{407}{500}}$

Fill in the chart at right that represents the sample space for the *sum* you get when you roll two six-sided dice.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Determine if the events below are *mutually exclusive* or *inclusive*.

$P(\text{sum of 6 } or \text{ sum greater than } 8) = P(6) + P(>8)$

mutually exclusive

$= \frac{5}{36} + \frac{10}{36} = \frac{15}{36} = \boxed{\frac{5}{12}}$

$P(\text{sum greater than 6 } or \text{ sum less than } 8) = P(>6) + P(<8) - P(>6 \text{ and } <8)$

inclusive

$= \frac{21}{36} + \frac{21}{36} - \frac{6}{36} = \frac{36}{36} = \boxed{1}$ → 100% chance of happening

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Review: Roll a regular six-sided die.

$P(4) = \frac{1}{6}$

$P(\text{not } 4) = \frac{5}{6}$

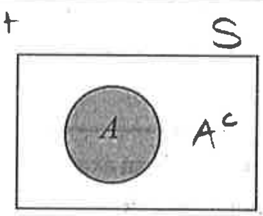
$P(4) + P(\text{not } 4) = \frac{1}{6} + \frac{5}{6} = \frac{6}{6} = \boxed{1}$

In general, $P(A) + P(A^c) = 1$

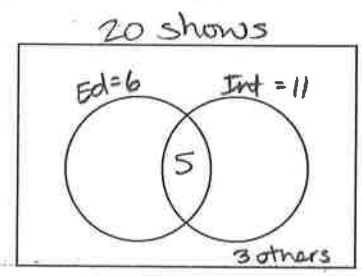
* A^c = complement of A, not A

or $P(A) = 1 - P(A^c)$

or $P(A^c) = 1 - P(A)$



Students were asked to decide which TV shows to watch choosing from 20 shows.
Of the 20 shows, 6 are educational, 11 are interesting and 5 are both educational and interesting.
Draw a diagram to illustrate the sample space. [3 others]
Are these events *mutually exclusive* or *inclusive?*



Find the following probabilities. Write your answer as a *fraction* and as a *percent*.

$P(\text{educational}) = \frac{6}{20} = \frac{3}{10} = \boxed{30\%}$

$P(\text{interesting}) = \frac{11}{20} = \boxed{55\%}$

$P(\text{educational } or \text{ interesting}) = P(E) + P(I) - P(E \,\&\, I)$

inclusive

$= \frac{6}{20} + \frac{11}{20} - \frac{5}{20} = \frac{12}{20} = \frac{3}{5} = \boxed{60\%}$

$P(\text{not educational}) = 1 - P(E) = 1 - \frac{3}{10} = \frac{7}{10} = \boxed{70\%}$

$P(\text{not interesting}) = 1 - P(I) = 1 - \frac{11}{20} = \frac{9}{20} = \boxed{45\%}$

When do you multiply to find the probability?

Suppose you have two events: one is to toss a coin, the other to roll a six-sided die.

Use the FCP to determine the total number of different outcomes: $\underset{\text{coin}}{2} \cdot \underset{\text{die}}{6} = 12$.

List the different outcomes.

H1 H2 H3 H4 H5 H6

T1 T2 T3 T4 T5 T6

Find: $P(H) = \frac{6}{12} = \boxed{\frac{1}{2}}$ $P(3) = \frac{2}{12} = \boxed{\frac{1}{6}}$ $P(H \text{ and } 3) = \boxed{\frac{1}{12}}$ $\longrightarrow$ $P(H) \cdot P(3) = \frac{1}{2} \cdot \frac{1}{6} = \boxed{\frac{1}{12}}$

Conclusion: $P(H \text{ and } 3) = P(H) \cdot P(3)$

What happens on the coin is independent (has no effect) on what happens to the die.

Independent Events

In general, events A and B are independent $\Leftrightarrow P(\text{A } and \text{ B}) = P(A) \cdot P(B)$

15 total 8 total

Example: Michelle has 6 quarters, 4 dimes, and 5 pennies in her pocket. Darren has 2 nickels, 1 quarter, and 5 pennies in his pocket. If Michelle selects a coin at random out of her pocket, will that have any effect on a coin that Darren selects at random out of his pocket? NO These events are independent.

$P(\text{Michelle selects a dime } and \text{ Darren selects a penny}) = P(M \text{ dime}) \cdot P(D \text{ penny})$

$= \frac{\cancel{4}}{3\,\cancel{15}} \cdot \frac{\cancel{5}\,1}{\cancel{8}\,2} = \boxed{\frac{1}{6}}$

$P(\text{they each pick out a penny}) = P(M \text{ penny}) \cdot P(D \text{ penny})$

$= \frac{5}{3\,\cancel{15}} \cdot \frac{\cancel{5}\,1}{8} = \boxed{\frac{5}{24}}$

$P(\text{they each pick out a dime}) = P(M \text{ dime}) \cdot P(D \text{ dime})$

$= \frac{4}{15} \cdot 0 = \boxed{0} \rightarrow$ impossible

$P(\text{Michelle does } \underline{\text{not}} \text{ pick a penny } and \text{ Darren does } \underline{\text{not}} \text{ pick a nickel}) = \frac{2\,\cancel{10}}{3\,\cancel{15}} \cdot \frac{\cancel{6}\,3}{\cancel{8}\,4} = \frac{6}{12} = \boxed{\frac{1}{2}}$

Example: Suppose I have a bag with 1 red marble and 3 blue marbles.

If I withdraw a marble and $\boxed{\text{replace}}$ it in the bag, and then withdraw another marble, are these events *independent*?

How many outcomes are in my sample space? $4 \cdot 4 = \boxed{16}$

$P(\text{R } and \text{ B}) = P(R) \cdot P(B) = \frac{1}{4} \cdot \frac{3}{4} = \boxed{\frac{3}{16}}$

$P(\text{R } and \text{ R}) = P(R) \cdot P(R) = \frac{1}{4} \cdot \frac{1}{4} = \boxed{\frac{1}{16}}$

$P(\text{B } and \text{ B}) = P(B) \cdot P(B) = \frac{3}{4} \cdot \frac{3}{4} = \boxed{\frac{9}{16}}$

If I withdraw a marble and **don't** replace it in the bag, and then withdraw another marble, are these events *independent*? $\boxed{\text{NO}}$

How many outcomes are in my sample space? $4 \cdot 3 = \boxed{12}$

only 3 left

$P(\text{R } and \text{ B}) = \frac{1}{4} \cdot \frac{3}{3} = \boxed{\frac{1}{4}}$

# of marbles changes

$P(\text{R } and \text{ R}) = \frac{1}{4} \cdot \frac{0}{3} = \boxed{0}$

$P(\text{B } and \text{ B}) = \frac{3}{4} \cdot \frac{2}{3} = \boxed{\frac{1}{2}}$

If what happens on the first event *does* effect the outcome of the second event, the events are said to be dependent.

Dependent Events

In general, events A and B are dependent $\Leftrightarrow P(\text{A } and \text{ B}) = P(A) \cdot P(B|A)$

where $P(\text{B}|\text{A}) =$ prob of B if A has occurred (or is "given")

Date ________________ Notes 11.6 Analyzing Data

Measures of Central Tendency

The mean (denoted $\bar{X}$) of a data set is the average. → $\frac{\text{sum of data values}}{\text{\# of data values}}$

The median of a data set is the middle value when the data values are arranged in ascending or descending order. If there is an even number of values, then the median is the mean of the two middle values.

The mode of a data set is the value that is occurs most often. There can be one, more than one, or no mode.

Example 1: Find the mean, median, and mode for the data values: ~~25~~, 33, 30, ~~26~~, 28, 28, 29

mean = $\frac{25+33+30+26+28+28+29}{7}$

$= \boxed{199}$

median = $\boxed{28}$

25, 26, 28, 28, 29, 30, 33

mode = $\boxed{28}$

Example 2: The yearly bonuses for five managers are \$90,000, \$85,000, \$100,000, \$0, and \$80,000. Find the mean, median and mode explain which measures are most representative.

mean = $\frac{\$90{,}000 + \$85{,}000 + \$100{,}000 + \$0 + \$80{,}000}{5}$

$= \boxed{\$71{,}000}$

median = $\boxed{85}$

0, 80, 85, 90, 100

mode = $\boxed{\text{none}}$

The mean is not a good measure of central tendency because 4 data values are above the mean, and only one is below.

The median is a good measure of central tendency because there are 2 data values above and below

Frequency Table: a table that lists the number of times (or frequency) that each data value occurs.

Make a table for the data below on the number of books read by 24 students last month. Use your frequency table to find the mean.

4, 5, 2, 6, 1, 2, 4, 2, 2, 1, 1, 3, 1, 3, 2, 1, 2, 2, 4, 1, 1, 1, 4, 5

# of books	Freq.	Product
1	8	8
2	7	14
3	2	6
4	4	16
5	2	10
6	1	6
Total	24	60

$\bar{x} = \frac{\text{product total}}{\text{frequency total}}$

$= \frac{60}{24} = \boxed{2.5}$

Make a histogram (bar graph).