Algebra 2
Ch. 1.2

Name: Key

1.2 – Properties of Real Numbers

Objective: To graph & order real numbers. To identify properties of real numbers.

Know the following **vocabulary**. Complete the diagram that illustrates the relationships between the different number systems.

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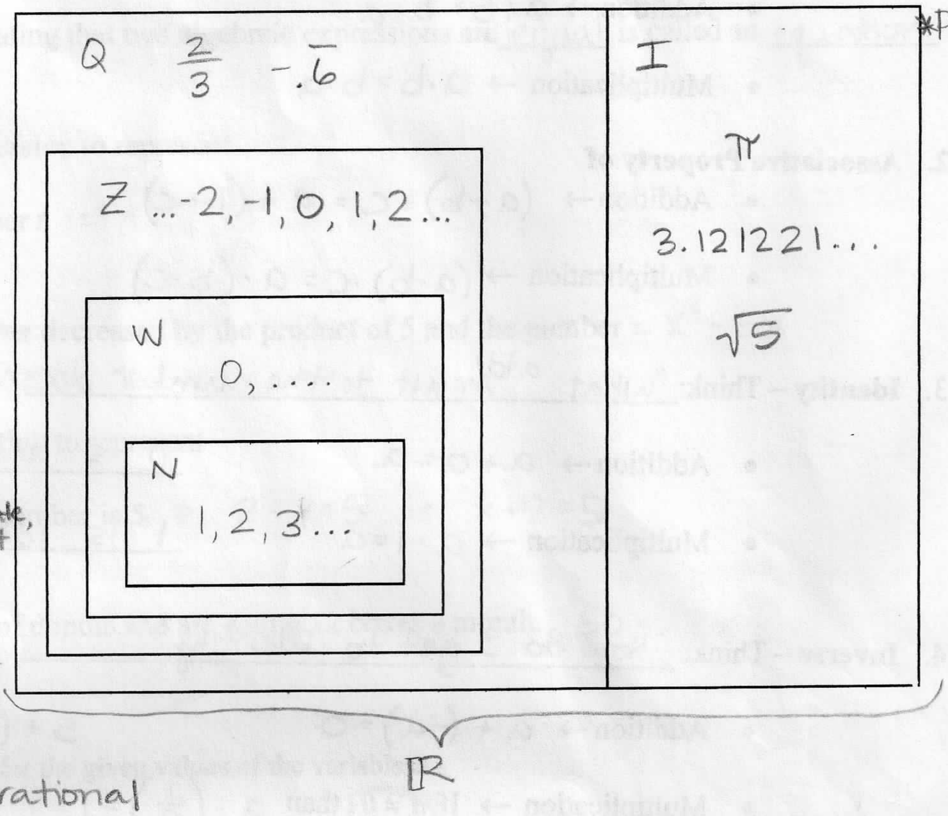

N ***Natural or Counting Numbers:***

1, 2, 3...

W ***Whole Numbers:***

0, 1, 2, 3...

Z ***Integers:***

...-2, -1, 0, 1, 2...

Q ***Rational Numbers:*** $\frac{P}{Q}$ → terminate, repeat

$\frac{2}{3}$, 0.32

I ***Irrational Numbers:***

non-terminating, non repeat

π, $\sqrt{2}$

ℝ ***Real Numbers:***

All rational and irrational

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Classify **in as many ways as possible:**

56	5.6	$5.\overline{6}$	5.616616661...
N, W, Z, Q, ℝ	Q, ℝ	Q, ℝ	I, ℝ

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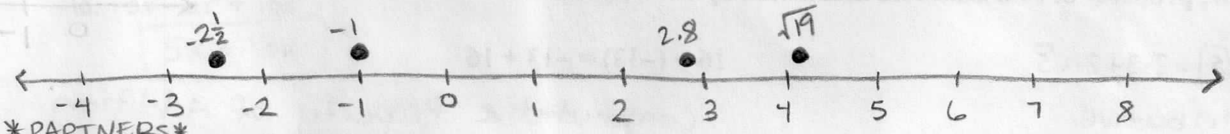
Graph each number on a number line.

-1 $\quad \sqrt{19}$ $\quad 2.8$ $\quad -2\frac{1}{2}$



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Compare the two numbers. Use > or <.

$-\sqrt{2}$ __>__ -2 $\qquad$ 4 __<__ $\sqrt{17}$ $\qquad$ $\sqrt{29}$ __>__ 5 $\qquad$ $\sqrt{50}$ __>__ 6.8

Properties of Real Numbers

For any real numbers a, b, and c — Examples

1. Commutative Property of

- Addition→ $a+b=b+a$ — $3+4=4+3$
- Multiplication → $a \cdot b = b \cdot a$ — $4 \cdot 3 = 3 \cdot 4$

2. Associative Property of

- Addition→ $(a+b)+c = a+(b+c)$ — $(3+4)+5 = 3+(4+5)$
- Multiplication → $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ — $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$

3. **Identity** – Think: "What add/mult to the number doesn't change the value"

- Addition→ $a+0=a$ — "0" is identity for addition
- Multiplication → $a \cdot 1 = a$ — "1" is identity for multiplication

4. **Inverse** – Think: "How do I get to Identity?"

- Addition→ $a+(-a)=0$ — $5+(-5)=0$
- Multiplication → If $a \neq 0$, than $a \cdot \left(\frac{1}{a}\right)=1$ — $5 \cdot \left(\frac{1}{5}\right)=1$

$-\frac{2}{3} \cdot \left(-\frac{3}{2}\right)=1$

5. **Distributive Property** → If $a(b+c)$ then $ab+ac$ and $(a+c)b = ab+bc$

Identify the opposite or additive inverse, and the reciprocal or multiplicative inverse, for each number

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7 Add inverse: -7 Mult inverse: $\frac{1}{7}$

-4.5 ($-\frac{9}{2}$) Add: 4.5 Mult: $-\frac{2}{9}$

$\frac{3}{4}$ Add: $-\frac{3}{4}$ Mult: $\frac{4}{3}$

$-\frac{6}{7}$ Add: $\frac{6}{7}$ Mult: $-\frac{7}{6}$

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Name the property of real numbers illustrated by each equation.

$2(3+\sqrt{5}) = 2 \cdot 3 + 2 \cdot \sqrt{5}$

Distributive

$16+(-13) = -13+16$

Commutative Property of Addition

Using Real Properties to Simplify Expressions

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1. Simplify $4(3a-b)+2(b+3a)$

$12a-4b+2b+6a$

$= \boxed{18a-2b}$

2. Simplify $2(3x-y)-4(2x+3y)$

$6x-2y-8x-12y$

$= \boxed{-2x-14y}$

Section 1.3 Algebraic Expressions

Objective: To evaluate and to simplify algebraic expressions
MP: Reason abstractly and quantitatively

I. Verbal Expressions to Algebraic Expressions

A mathematical sentence containing one or more variables is called an open sentence. $3x+4$

A mathematical sentence stating that two algebraic expressions are equal is called an equation $3x+4=12$

Example: *PARTNERS*

1. Write an algebraic expression to represent

   a. 7 less than the number t $= t-7$

   b. the square of a number decreased by the product of 5 and the number $= x^2 - 5x$

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2. Write an algebraic equation to represent

   a. The sum of 2 and a number is 5 $=$ $2+x=5$ or $x+2=5$

   b. You have 50 boxes of donuts and are eating 12 boxes a month. $50 = 12x$

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3. Evaluate each expression for the given values of the variables.

   a. $-4v + 3(w + 2v) - 5w$; $v = -2$ and $w = 4$

   $-4(-2) + 3(4) + 2(-2)) - 5(4)$ $\quad 8 - 20 = \boxed{-12}$

   $8 + 3(4-4) - 20$

   $8 + 3(0) - 20$

   b. $c(3 - a) - c^2$; $a = 4$ and $c = -1$

   $(-1)(3-(4)) - (-1)^2$

   $-1(-1) - (1)$

   $1 - 1 = \boxed{0}$

4. Simplify by combining like terms

   a. $5x - 3x^2 + 16x^2$

   $\boxed{5x + 13x^2}$

   b. $\dfrac{3(a-b)}{9} + \dfrac{4}{9}b$

   $\dfrac{3a-3b}{9} + \dfrac{4b}{9}$

   $\dfrac{3a-3b+4b}{9}$

   $\dfrac{3a+b}{9}$

   $\dfrac{3a}{9} + \dfrac{b}{9}$

   $\boxed{\dfrac{a}{3} + \dfrac{b}{9}}$

Section 1.4 – Solving Equations

Objective: To solve equations and to solve problems by writing equations.
MP: reason abstractly and quantitatively

Verbal Expressions to Algebraic Expressions

Write an algebraic equation to represent

a. The sum of 2 and a number is 5 $2+x=5$ or $x+2=5$

b. The difference of 2 and three times a number is 11 $11=2-3x$

Properties of Equality **Examples**

- Reflexive → For any real number a, $a=a$ $4=4$
- Symmetric → For all real numbers a and b
  If $a=b$, then $b=a$ $x=4$ or $4=x$
- Transitive → For all real numbers a, b, and c
  If $a=b$ and $b=c$, then $a=c$ $x=y$; $y=4$, then $x=4$
- Substitution → If $a = b$, then a may be replaced by b and b may be replaced by a.

Example: $2a+4b=3b+6$; $a=2$ $2a+4a=3a+6$

$2(2)+4(2)=3(2)+6$

$4+8=6+6$

PARTNERS Name the property illustrated by each statement

1. $a-2.03=a-2.03$ Reflexive

2. If $3=x$ and $x=y$, then $3=y$ Transitive

More Properties of Equality **Examples**

- Addition → $If\ a=b, then\ a+c=b+c$ if $x=12$, then $x+3=12+3$
- Subtraction → $If\ a=b, then\ a-c=b-c$ if $x=12$, then $x-3=12-3$
- Multiplication → $If\ a=b, then\ a\cdot c=b\cdot c$ if $x=12$, then $x\cdot 3=12\cdot 3$
- Division → $If\ a=b, then\ a\div c=b\div c\ (with\ c\neq 0)$ if $x=12$, then $x\div 3=12\div 3$

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Use the properties of you have learned so far to solve for unknown variables below

3. $s-5.48=0.02$

$+5.48\ \ +5.48$

$\boxed{s=5.5}$

4. $\frac{3}{2}\cdot 18=\frac{2}{3}t\cdot\frac{3}{2}$

$\frac{3}{\cancel{2}}\cdot\frac{\overset{9}{\cancel{18}}}{1}=\frac{\cancel{2}}{\cancel{3}}t\cdot\frac{\cancel{3}}{\cancel{2}}$

$\boxed{27=t}$

5. $53=3(y-2)-2(3y-1)$

$53=3y-6-6y+2$

$53=-3y-4$

$+4\ \ \ \ +4$

$\frac{57}{-3}=\frac{-3y}{-3}$

$\boxed{-19=y}$

Equations with No Solutions and Identities

Is the equation $11+3x-7=6x+5-3x$ *always, sometimes, or never* true?

$3x+4=3x+5$

$-3x \quad -3x$

$4=5$

$\boxed{\text{Never}}$

Is the equation $6x+5-2x=4+4x+1$ *always, sometimes, or never* true?

$4x+5=4x+5$

$-4x \quad -4x$

$5=5$

$\boxed{\text{Always}}$

Solving a Literal Equation

The equation $C=\frac{5}{9}(F-32)$ relates temperatures in degrees Fahrenheit F and degrees Celsius C. What is F in terms of C?

$\frac{9}{5}\cdot C=\frac{5}{9}(F-32)\cdot\frac{9}{5}$

$\frac{9}{5}C=F-32$

$+32 \quad +32$

$\boxed{F=\frac{9}{5}C+32}$

* Properties of Inequalities

Section 1.5 – Solving Inequalities

Objective: To solve and graph inequalities. To write and solve compound inequalities
MP: Make sense of problems + persevere in solving them.

For any two real numbers, *a* and *b*, exactly one of the following statements is true:

$a < b$ $a = b$ $a > b$

Inequalities can be written the following ways:

$<$ - less than $\leq$ - less than or equal to

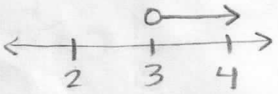
$>$ - greater than $\geq$ - greater than or equal to

Solution - set notation
$\{x| \quad \}$
ex: $x > 4$ $\{x | x > 4\}$
The set of all x such that x is greater than 4

Inequalities are solved using the <u>same process</u> as solving equations. Here are some additional tips:

1. Reverse the inequality when you
   $3x + 4 > 7$
   solve like $3x + 4 = 7$
   - Multiply or Divide by a negative
2. When do you use an *open dot* vs. a *solid dot* when graphing an inequality?
   - open dot - symbols $<$ or $>$ → "Don't include"
   - Solid dot - symbols $\leq$ or $\geq$ → "Include"
3. Which direction is the ray?
   - The ray (shaded region) points in direction inequality points, if variable is on left side.

ex: $x > 3$



Examples: *Partners*

Write the inequality that represents the sentence.

1. Four less than a number is greater than –28. $\boxed{n - 4 > -28}$
2. Twice a number is at least 15. $\boxed{2n \geq 15}$
3. A number increased by 7 is less than 5. $\boxed{n + 7 < 5}$
4. The quotient of a number and 8 is at most –6. $\boxed{\frac{n}{8} \leq -6}$

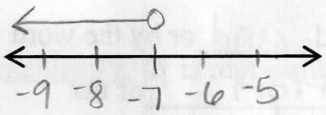
Solve each inequality. Graph the solution.

1. $2w + 19 < 5$

$-19 \quad -19$

$\frac{2w}{2} < \frac{-14}{2}$

$\boxed{w < -7}$



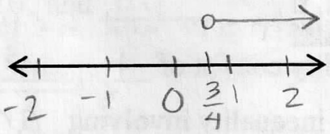
2. $11 \cdot \frac{3z+6}{11} < z \cdot 11$

$3z + 6 < 11z$

$-3z \quad -3z$

$\frac{6}{8} < \frac{8z}{8}$

$\frac{3}{4} < z \rightarrow \boxed{z > \frac{3}{4}}$



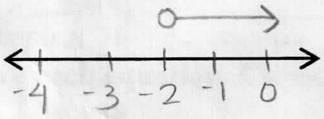
3. $2y + 9 < 5y + 15$

$-5y \quad -5y$

$-3y + 9 < +15$

$-9 \quad -9$

$\frac{-3y}{-3} < \frac{6}{-3}$ $\boxed{y > 2}$



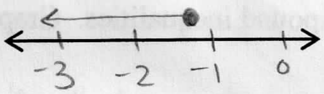
4. $5(3x+2) \le 4(x-1)$

$15x + 10 \le 4x - 4$

$-4x - 10 \quad -4x - 10$

$\frac{11x}{11} \le \frac{-14}{11}$

$\boxed{x \le \frac{-14}{11}} \rightarrow x \le -1.\ldots$



5. $-2(w-7) + 3 > w - 1$

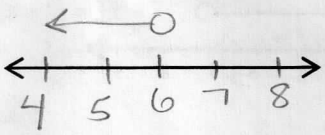
$-2w + 14 + 3 > w - 1$

$-2w + 17 > w - 1$

$-w \quad -17 \quad -w \quad -17$

$\frac{-3w}{-3} > \frac{-18}{-3}$

$\boxed{w < 6}$

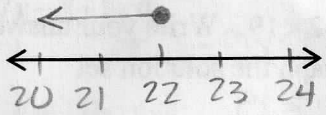


6. $3 \cdot \frac{1}{3}(7a-1) \le (2a+7) \cdot 3$

$7a - 1 \le 6a + 21$

$-6a + 1 \quad -6a + 1$

$\boxed{a \le 22}$



Using an Inequality

A movie rental company offers two subscription plans. You can pay \$36 a month and rent as many movies as desired, or you can pay \$15 a month and \$1.50 to rent each movie. How many movies must you rent in a month for the first plan to cost less than the second plan?

Let n = the # of movie rentals in one month

First plan: $= 36$

Second plan: $= 15 + 1.5n$

$36 < 15 + 1.5n$

$\frac{21}{1.5} < \frac{1.5n}{1.5}$

$14 < n$

You must rent more than 14 movies in a month for the first plan to cost less.

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No Solution or All Real Numbers as Solutions

Is the inequality always, sometimes, or never true? $-2(3x+1) > -6x+7$

$-6x-2 > -6x+7$
$+6x \quad\quad +6x$
$-2 > 7$

$\boxed{\text{False}}$ – never true

Is the inequality always, sometimes, or never true? $5(2x-3)-7x \le 3x+8$

$10x-15-7x \le 3x+8$ $\quad -15 \le 8$
$3x-15 \le 3x+8$

$\boxed{\text{Always True}}$

Compound Inequalities $h \ge 10$ and $h \le 16$

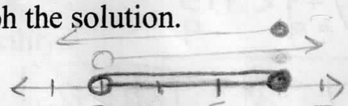
A **compound inequality** consist of __two inequalities__ joined by the word __and__ or by the word __or__

To solve a compound inequality involving __and__, find the __intersection__ of the __solution sets__ of the two inequalities. (conjunction) To solve a compound inequality involving __or__, find the __union__ of __solution sets__ of the two inequalities. disjunctions

Solve in Partners, graph together

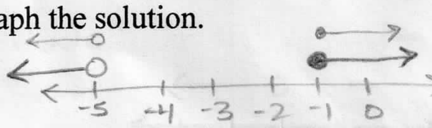
And Inequalities – What is the solution of $7 < 2x+1$ *and* $3x \le 18$? Graph the solution.

$6 < 2x$ $\quad$ $\frac{3x}{3} \le \frac{18}{3}$
$3 < x$ and $x \le 6$
$3 < x \le 6$



Or Inequalities – What is the solution of $7+k \ge 6$ *or* $8+k < 3$? Graph the solution.

$k \ge -1$ or $k < -5$

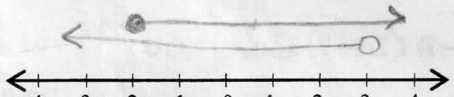


Partners

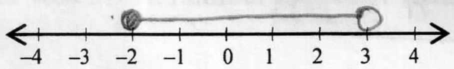
Examples: Solve the compound inequalities. Graph the solutions.

8. $x \ge -2$ *and* $x < 3$

Work:



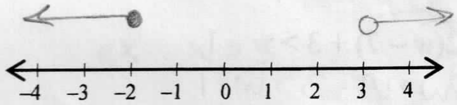
Answer:



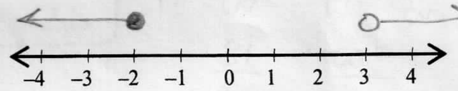
$-2 \le x < 3$

9. $x \le -2$ *or* $x > 3$

Work:



Answer:



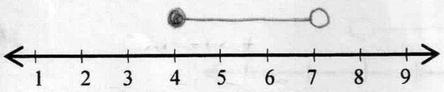
Together

10. Solve $10 \le 3y-2 < 19$. Write your answer in set notation. Graph the solution set

$10 \le 3y-2 < 19$
$+2 \quad\quad +2 \quad +2$
$\frac{12}{3} \le \frac{3y}{3} < \frac{21}{3}$

$4 \le y < 7$

$\{y \mid 4 \le y < 7\}$



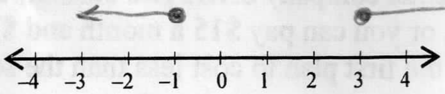
Partners

11. Solve $|2x-2| \ge 4$. Write your answer in set notation. Graph the solution set

$2x-2 \ge 4$ $\quad$ or $\quad$ $-(2x-2) \ge 4$
$+2 \quad +2$ $\quad\quad$ $-2x+2 \ge 4$
$\frac{2x}{2} \ge \frac{6}{2}$ $\quad\quad$ $\frac{-2x}{-2} \ge \frac{2}{-2}$
$x \ge 3$ $\quad$ or $\quad$ $x \le -1$

8

$\{x \mid x \ge 3 \text{ or } x \le -1\}$



Section 1.6 Solving Absolute Value Equations and Inequalities

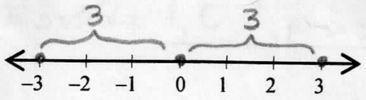
objective: To write and solve equations and inequalities involving absolute value.

I. Solving Absolute Value Equations

***The absolute value of a number is its distance from 0 on the number line
Since distance is positive, then absolute value is positive

Symbolically: For any real number *a*, $|a| = a$, if $a \geq 0$, and $|a| = -a$ (opposite of), if $a < 0$

Graphically, |3| is represented by



$|x| = 3 \rightarrow x = 3$ or $x = -3$

same as $|-3|$

Note:
The solution set for an equation that has no solution is the empty set symbolized by { } or $\varnothing$. Always make sure to check your solutions!

An extraneous solution is a solution derived from an original equation that is not a solution of the original equation.

You must isolate the absolute value expression before rewriting as two equations.

Examples:
Solve each equation. Check your answers.

1. $|-3x| = 18$

$-3x = 18$ or $-(-3x) = 18$
$\frac{-3x}{-3} = \frac{18}{-3}$ $3x = 18$
$\boxed{x = -6 \text{ or } x = 6}$ ✓

2. $|2x - 1| = 5$

$2x - 1 = 5$ or $-(2x-1) = 5$
$2x = 6$ $-2x + 1 = 6$
$\boxed{x = 3 \text{ or } x = -2}$ ✓

3. $3|z + 7| = 12$

$\frac{3|z+7|}{3} = \frac{12}{3}$
$z + 7 = 4$ or $-(z+7) = 4$
$\boxed{z = -3}$ $-z - 7 = 4$
or $\boxed{z = -11}$ ✓

4. $|4 - 2y| + 5 = 9$

$-5 \quad -5$
$4 - 2y = 4$ or $-(4-2y) = 4$
$-2y = 0$ $-4 + 2y = 4$
$+4 \quad +4$
$y = 0$ or $2y = 8$
$y = 4$

$|4 - 2y| = 4$
$4 - 2y = 4$ or $-(4-2y) = 4$
$-2y = 0$ $-4 + 2y = 4$
$y = 0$ $2y = 8$
$y = 4$

negatives can masquerade as positives

Solve each equation. Check for extraneous solutions.

5. $|x + 5| = 3x - 7$

$x + 5 = 3x - 7$ or $-(x+5) = 3x - 7$
$-x \; +7 \; -x \; +7$ $-x - 5 = 3x - 7$
$12 = 2x$ $+x \; +7 \; +x \; +7$
$\boxed{6 = x}$ or $2 = 4x$
$\cancel{x = \frac{1}{2}}$

6. $|7y - 3| + 1 = 0$

$7y - 3 = -1$ or $-(7y - 3) = -1$

empty set
no solution

(abs. val. can't be negative)

7. $|4w + 3| - 2 = 5$

$4w + 3 = 7$ or $-(4w + 3) = 7$
$-3 \; -3$ $-4w - 3 = 7$
$4w = 4$ $\frac{-4w}{-4} = \frac{10}{-4}$
$\boxed{w = 1}$ or $\boxed{w = -\frac{5}{2}}$

8. $2|z + 1| - 3 = z - 2$

$+3 \quad +3$
$\frac{2|z+1|}{2} = \frac{z+1}{2}$
$2(z + 1 = \frac{1}{2}z + \frac{1}{2})$ or $-(z+1) = \frac{1}{2}z + \frac{1}{2}$
$2z + 2 = z + 1$ $(-z - 1 = \frac{1}{2}z + \frac{1}{2})2$
$-z \; -2 \; -z \; -2$ $-2z - 2 = z + 1$
$\boxed{z = -1}$ $-z \; +2 \; -z \; +2$
or $-3z = 3$ $\boxed{z = -1}$

II. Solving Absolute Value Inequalities.

Describe the set of numbers that would make the following statement true: $|x| \le 2$

$x \le 2$ and $x \ge -2$

All real numbers between -2 and 2 including -2 and 2

Describe the set of numbers that would make the following statement true: $|x| > 1$

$x > 1$ or $x < -1$

Rewrite $|x| > a$: $x > a$ or $x < -a$ * more is or *

Rewrite $|x| \le a$: $x \le a$ and $x \ge -a$

You must isolate the Absolute Value expression before rewriting as two inequalities.

Solve each inequality. Graph the solution.

1. $|2t-3| \le 5$

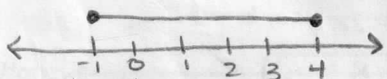
$2t-3 \le 5$ and $-(2t-3) \le 5$

$+3 \;\; +3$ $\quad -2t+3 \le 5$

$2t \le 8$ $\quad -3 \;\; -3$

$-2t \le 2$

$t \le 4$ and $t \ge -1$



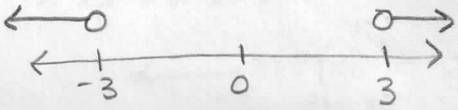
$\boxed{-1 \le x \le 4}$

2. $|4b| - 3 > 9$

$+3 \;\; +3$

$\frac{4b}{4} > \frac{12}{4}$ or $\frac{-4b}{-4} > \frac{12}{-4}$

$b > 3$ or $b < -3$



3. $\frac{5|y+3|}{5} < \frac{15}{5}$

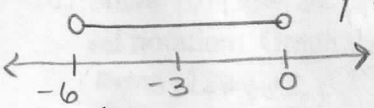
$y+3 < 3$ and $-(y+3) < 3$

$-3 \;\; -3$ $\quad -y-3 < 3$

$y < 0$ $\quad +3 \;\; +3$

$\frac{-y}{-1} < \frac{6}{-1}$

$y > -6$ $\quad \boxed{0 < x < -6}$



4. $2|4x+1| - 5 \le 1$

$+5 \;\; +5$

$\frac{2|4x+1|}{2} \le \frac{6}{2}$

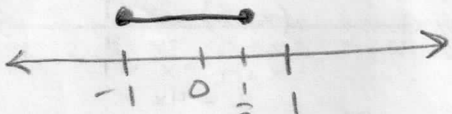
$4x+1 \le 3$ and $-(4x+1) \le 3$

$-1 \;\; -1$ $\quad -4x-1 \le 3$

$4x \le 2$ $\quad -4x \le 4$

$x \le \frac{1}{2}$ and $x \ge -1$

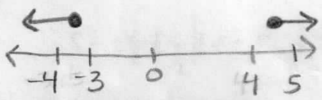
$\boxed{-1 \le x \le \frac{1}{2}}$



5. $\frac{1}{2}|2w-1| - 3 \ge 1$

$+3 \;\; +3$

$2 \cdot \frac{1}{2}|2w-1| \ge 4 \cdot 2$



$2w-1 \ge 8$ or $-(2w-1) \ge 8$

$2w \ge 9$ $\quad -2w+1 \ge 8$

$-2w \ge 7$

$w \ge \frac{9}{2}$ $\quad w \le -\frac{7}{2}$

$w \ge 4.5$ or $w \le -3.5$

6. $|-2x+4| \ge 4$

$-2x+4 \ge 4$ or $-(-2x+4) \ge 4$

$-2x \ge 0$ $\quad 2x - 4 \ge 4$

$2x \ge 8$

$x \le 0$ or $x \ge 4$