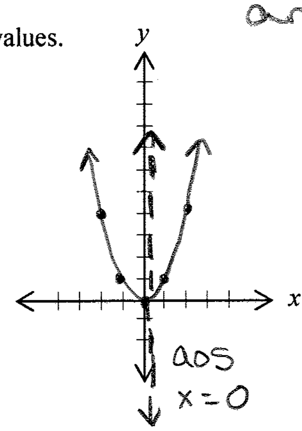
Date __________ Notes 4.1 Introduction of Quadratic Functions and Transformations

Graph $y = x^2$. Use a table of values.

x	y
-2	4
-1	1
0	0
1	1
2	4

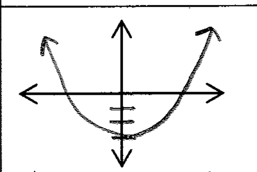
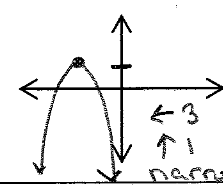
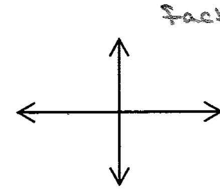
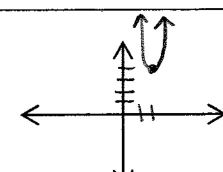


This is called a quadratic function

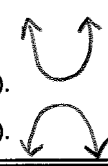
$f(x) = ax^2 + bx + c \quad (a \neq 0)$

Properties:

1. Has a max or min value which is the y-coordinate of the vertex.
2. Has an axis of symmetry. (aos)
3. Domain = $\{x : x \in \mathbb{R}\}$ always Range = $\{y : y \leq \max\}$ OR $\{y : y \geq \min\}$

Function	Is it a quadratic function?	Write in the form $f(x) = ax^2 + bx + c$ if possible	Graph the quadratic functions.	Does the parabola open up or down?	Is the y-coord. of the vertex a max or min? Find the vertex and AOS
$f(x) = \frac{1}{2}x^2 - 3$	yes	$f(x) = \frac{1}{2}x^2 - 3$ $a = 1/2 \quad b = 0 \quad c = -3$	 ↓3 wider by factor of 1/2	up	minimum of -3 V (0, -3) aos x = 0
$g(x) = -2(x+3)^2 + 1$	yes	$g(x) = -2(x^2+6x+9)+1$ $g(x) = -2x^2-12x-18+1$ $g(x) = -2x^2-12x-17$ $a=-2 \quad b=-12 \quad c=-17$	 ←3 ↑1 narrow by factor of 2	down	max of 1 V(-3, 1) aos x = -3
$h(x) = -4x(x^2+5)$ $h(x) = -4x^3 - 20$	NO				
$j(x) = 3(x-2)^2 + 4$	yes	$j(x) = 3(x^2-4x+4)+4$ $j(x) = 3x^2-12x+12+4$ $j(x) = 3x^2-12x+16$ $a=3 \quad b=-12 \quad c=16$	 →2 ↑4 narrow by factor of 3	up	min of 4 V (2, 4) aos x = 2

Conclusion: Given $f(x) = ax^2 + bx + c$, where $a \neq 0$.



1. The parabola opens up when $a > 0$. The y-coordinate of the vertex is a min (max or min).
2. The parabola opens down when $a < 0$. The y-coordinate of the vertex is a max (max or min).

Without graphing, determine whether the parabola $f(x) = -2(3x+5)(6-x)$ opens up or down and whether the y-coord. of the vertex is a maximum or a minimum. [Hint: what do you need to find?] "a"

Opens up or down? up

Has a max or min? min

$f(x) = -2(18x - 3x^2 + 30 - 5x)$ $\boxed{a = 6}$

$f(x) = -2(-3x^2 + 13x + 30)$

$f(x) = \boxed{6x^2} - 26x - 60$

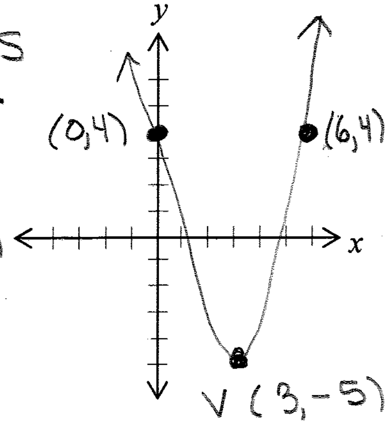
Graph $y=(x-3)^2-5$. The vertex is $(3,-5)$.

$y = x^2-6x+9-5$
$y = x^2-6x+4$

$b=4$ (y int)

State the transformations from the parent graph $y=x^2$.

Horizontal shift right 3
Vertical shift down -5



x	y
0	4
3	-5
6	4

Symmetry!

The form $y=(x-3)^2-5$ is called the Vertex form of a parabola because you can easily identify the vertex.

Vertex Form

A quadratic function, $y=ax^2+bx+c$ with $a\neq 0$, can be written in vertex form:

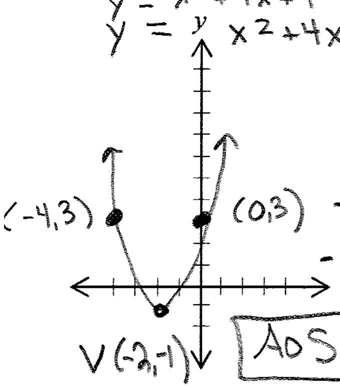
$y=a(x-h)^2+k$ vertex (h,k)

Axis of symmetry: $x=h$

Graph each function. Describe how it was translated from $f(x)=x^2$. Identify the axis of symmetry.

1. $y=(x+2)^2-1$

$y=x^2+4x+4-1$
$y=x^2+4x+3$



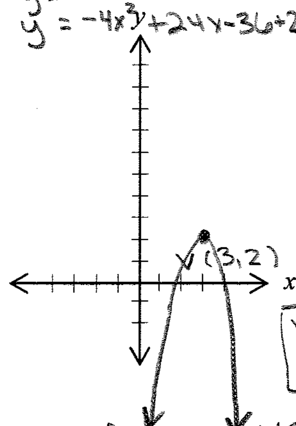
x	y
0	3
-2	-1
-4	3

$\boxed{\text{AOS } x=-2}$

Horizontal Shift Left 2
Vertical Shift Down 1

2. $y=-4(x-3)^2+2$ $y=-4x^2+24x-34$

$y=-4(x^2-6x+9)+2$
$y=-4x^2+24x-36+2$

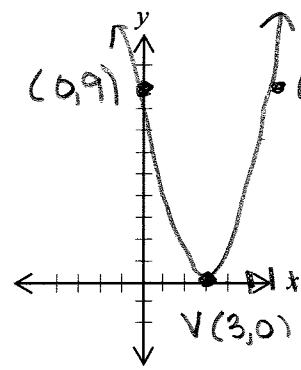


x	y
0	-34
3	2
6	-34

$\boxed{\text{AOS } x=3}$

HS Right 3, VS up 2,
Reflected over x-axis, V. Stretch by factor of 4

3. $f(x)=(x-3)^2$ $f(x)=x^2-6x+9$



x	y
0	9
3	0
6	9

HS right 3

$\boxed{\text{AOS } x=3}$

Identify the vertex, axis of symmetry, the maximum or minimum value, and the domain and the range of each function.

1. $y=(x-2)^2+3$ opens up

V: $(2,3)$ aos $x=2$

min of 3

D: $\{x \mid x\in\mathbb{R}\}$

R: $\{y \mid y\geq 3\}$

2. $f(x)=-0.2(x+3)^2+2$ opens down

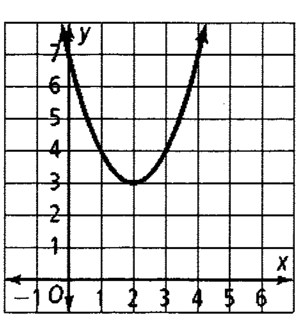
V: $(-3,2)$ aos $x=-3$

max of 2

D: $\{x \mid x\in\mathbb{R}\}$

R: $\{y \mid y\leq 2\}$

Write a quadratic function to model the graph.



1. V $(2,3)$
2. $y=a(x-2)^2+3$
3. Choose $(0,7)$

$7=a(0-2)^2+3$
$7=a(4)+3$
$4=4a$
$1=a$

4. $y=1(x-2)^2+3$

$\boxed{y=(x-2)^2+3}$

1. Find the vertex
2. Plug in to $f(x)=a(x-h)^2+k$
3. Choose another point and plug in for x and y.
4. Solve for a.
5. Write your final equation in vertex form

Date _______________ Notes 4.2: Standard Form of a Quadratic Function

Standard Form of a Quadratic Equation: $y = ax^2 + bx + c$

Shortcut to find the Vertex:

Equation for axis of symmetry: $X = \frac{-b}{2a}$

x-value of vertex = $\frac{-b}{2a}$

How can you find the corresponding y-value of the vertex?

Plug in the x-value of the vertex $x = \frac{-b}{2a}$ into the original equation to find y

Notation

Summary - Quadratic Formula in standard form

- The graph $f(x) = ax^2 + bx + c, a \neq 0$ is a parabola
- If $a > 0$, the parabola opens up (min). If $a < 0$, the parabola opens down (max)
- The axis of symmetry is the line $x = \frac{-b}{2a}$
- The x coordinate of the vertex is $\frac{-b}{2a}$. The y coordinate of the vertex is $y = f\left(-\frac{b}{2a}\right)$.
- The y-intercept is $(0, c)$
  Let $x = 0$

Use the shortcut to write the equation for the axis of symmetry and find the coordinates of the vertex.

a) $g(x) = x^2 - 4x + 1$

$x = \frac{-b}{2a}$ (x coordinate of vertex and aos)

$x = \frac{-(-4)}{2(1)} = 2$

$g(x) = x^2 - 4x + 1$

$g(2) = 2^2 - 4(2) + 1$
$= 4 - 8 + 1$
$= -4 + 1 = -3$

Vertex $(2, -3)$
AOS $x = 2$

b) $f(x) = 5x^2 - 10x + 4$

$x = \frac{-(-10)}{2(5)} = \frac{10}{10} = 1$

$f(1) = 5(1)^2 - 10(1) + 4$
$= 5 - 10 + 4$
$= -5 + 4 = -1$

Vertex $(1, -1)$
AOS $x = 1$

Write each function in vertex form.

1. $y = x^2 - 8x + 19$

- Identify a and b. $a = 1$, $b = -8$
- Find the x-coordinate of the vertex. $x = \frac{-(-8)}{2(1)} = \frac{8}{2} = 4$
- Substitute your answer into the equation for x. Solve for y.
  $y = 4^2 - 8(4) + 19$ $y = 16 - 32 + 19$ $y = -16 + 19$ $y = 3$
- The vertex is (4 , 3).
- Substitute h, k, and a into $y = a(x-h)^2 + k$.
  (The a is the same in standard form and vertex form) $\boxed{y = (x-4)^2 + 3}$

2. $y = x^2 - 2x - 6$

$x = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

$\boxed{y = (x-1)^2 - 7}$

$y = 1^2 - 2(1) - 6$
$= 1 - 2 - 6 = -7$

3. $y = 3x^2 + 12x + 5$

$a = 3$

$x = \frac{-12}{2(3)} = -2$

$\boxed{y = 3(x+2)^2 - 7}$

$y = 3(-2)^2 + 12(-2) + 5$
$y = 3(4) - 24 + 5 = -7$

Identify the vertex, the axis of symmetry, the maximum or minimum value, and the range of each parabola.

1. $y = 3x^2 + 18x + 32$

V : (-3, 5)
AOS x = -3
R : {y | y ≥ 5}

$x = \frac{-18}{2(3)} = -3$

$y = 3(-3)^2 + 18(-3) + 32$
$y = 3(9) - 54 + 32$
$y = 27 - 54 + 32$
$y = -27 + 32$
$y = 5$

opens up

2. $y = -x^2 + 2x + 3$

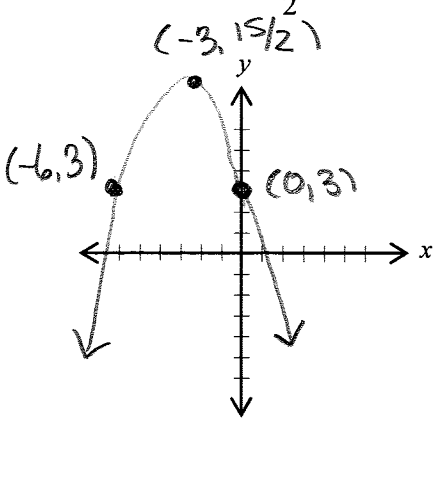
V : (1, 4)
AOS x = 1
R : {y | y ≤ 4}

$x = \frac{-2}{2(-1)} = 1$

$y = -1^2 + 2(1) + 3$
$y = -1 + 2 + 3 = 4$

opens down

Sketch: $y = -\frac{1}{2}x^2 - 3x + 3$



Steps:

1. **Find the Vertex**
2. **Find the y-intercept**
3. **Find the 3rd point on the graph (use notion of symmetry)**

③

x	y
0	3
-3	15/2
-6	3

Symmetry

① $x = \frac{-(-3)}{2(-\frac{1}{2})} = \frac{3}{-1} = -3$

$y = -\frac{1}{2}(9) + 9 + 3$

$y = -\frac{9}{2} + 9 + 3$

$y = 12 - 4\frac{1}{2}$
$y = 7\frac{1}{2}$
$y = 15/2$

Vertex (-3, 15/2)

② y int (0, 3)

Date ________________ Notes 4.4 Factoring

Factoring Flow Chart

Greatest Common Factor

2 terms Difference of Squares

3 terms Trial and Error (special case Perfect Square Trinomial)

4 terms Grouping

★Also Sum and Difference of Cubes (Chap 5)

Repeat if Possible

I. Factor out the GCF **Always check for this first!**

1. $27c^2 - 9c$

$\boxed{9c(3c-1)}$

2. $5x^3 + 25x^2 - 65x$

$\boxed{5x(x^2+5x-13)}$

3. $5z(2z+1) - 2(2z+1)$

$\boxed{(2z+1)(5z-2)}$

II. Two Terms

Comes from

Difference of Two Squares: $a^2 - b^2 =$ $(a+b)(a-b)$

$(a+b)(a-b) = a^2 - \cancel{ab} + \cancel{ab} - b^2 = a^2 - b^2$

1. $x^2 - 25$

$\boxed{(x+5)(x-5)}$

2. $16x^2 - 49$

$\boxed{(4x+9)(4x-9)}$

3. $8x^3 - 18x$

GCF first always if you can!

$2x(4x^2-9)$

$\boxed{2x(2x+3)(2x-3)}$

Don't drop this

4. $y^4 - 81$

$(y^2+9)(y^2-9)$

$\boxed{(y^2+9)(y+3)(y-3)}$

This is a sum of squares not a difference!

Squares

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144...

$x^2, x^4, x^6, x^8...$

(even powers are squares)

III. Three Terms Trial + Error **(the reverse of FOIL)**

Check Answer By FOIL

Sign Rules:

- If the sign on the third term is positive, you will have ++ or −− depending on the sign of the middle term.
- If the sign on the third term is negative, you will have +− in the parentheses.

1. $x^2+12x+27$

$\boxed{(x+9)(x+3)}$

2. $x^2-15x-54$

$\boxed{(x-18)(x+3)}$

3. $2x^2+10x-12$

$2(x^2+5x-6)$

$\boxed{2(x+6)(x-1)}$

4. $5x^2+14x+8$

$\boxed{(5x+4)(x+2)}$

5. $6x^2-11x+3$

$\boxed{(3x-1)(2x-3)}$

6. $8x^2+27x-20$

$(\quad)(\quad)$

Special Case

Perfect-Trinomial Squares: $a^2-2ab+b^2=$ $(a-b)^2$

$a^2+2ab+b^2=$ $(a+b)^2$

Comes from:

$(a-b)(a-b)=a^2-ab-ab+b^2$

$a^2-2ab+b^2$

$(a+b)(a+b)=a^2+ab+ab+b^2$

$a^2+2ab+b^2$

1. $x^2+10x+25$

$2\cdot5\cdot x$; x, 5 ✓ PST

$(x+5)^2$

2. $16x^2-40x+25$

$2\cdot4x\cdot5$; $4x$, 5 ✓ PST

$(4x-5)^2$

3. $36x^2-60x+25$

$2\cdot6x\cdot5$ ✓ ; $6x$, 5

$(6x-5)^2$

4. $3x^2-36x+108$

$3(x^2-12x+36)$

$3(x-6)^2$

IV. Four Terms Grouping

Steps:

- Group terms 1 and 2, group terms 3 and 4
- Factor a GCF from each group
- Factor out the common group

1. $\underbrace{3x^3-6x^2}+\underbrace{4x-8}$

$3x^2(x-2)+4(x-2)$

$\boxed{(x-2)(3x^2+4)}$

2. $10x^3-15x^2-2x+3$

$5x^2(2x-3)-(2x-3)$

$\boxed{(2x-3)(5x^2-1)}$

Date:______________ Notes 4.5 Quadratic Equations, ZPP

Methods to Solving Quadratic Equations:

1. **Zero Product Property** (ZPP) 4.5
2. **Square Root Theorem** 4.6
3. **Complete the Square** 4.6
4. **Quadratic Formula** 4.7

Steps:
1) Set equation = 0
2) Factor
3) Set each factor = 0
4) Solve

Zero Product Property: If $a \cdot b = 0 \Rightarrow a = 0$ or $b = 0$

Solve each equation by factoring and applying ZPP. Show all steps.

1. $x^2 - 16x = 0$

$x(x-16)=0$

$\boxed{x=0}$ $\quad x-16=0$ $\quad \boxed{x=16}$

2. $x^2 - 14x + 45 = 0$

$(x-9)(x-5)=0$

$x-9=0$ $\quad x-5=0$

$\boxed{x=9}$ $\quad \boxed{x=5}$

3. $3x^2 + 6x = -3$

$3x^2+6x+3=0$

$3(x^2+2x+1)=0$

$3(x+1)(x+1)=0$

$3\neq 0$ $\quad x+1=0$ $\quad x+1=0$

$\boxed{x=-1}$ $\quad \boxed{x=-1}$

Writing Equations from Roots:

A **root** of an equation is a value that makes the equation true.

Use the Zero Product Property to write a quadratic equation with each pair of values as roots:

1. 5 and 3

$x=5$ $\quad x=3$

$x-5=0$ $\quad x-3=0$

$(x-5)(x-3)=0$

$x^2-3x-5x+15=0$

$\boxed{x^2-8x+15=0}$

2. -4 and 4

$x=-4$ $\quad x=4$

$x+4=0$ $\quad x-4=0$

$(x+4)(x-4)=0$

$x^2-4x+4x-16=0$

$\boxed{x^2-16=0}$

Connection to Parabola:

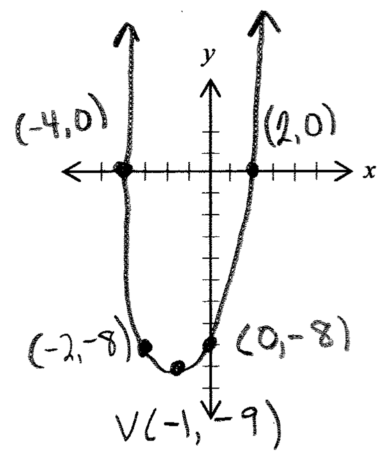
Graph $g(x) = x^2 + 2x - 8$.

$x = \frac{-2}{2(1)} = -1$

$g(-1)=(-1)^2+2(-1)-8$

Vertex $(-1,-5)$

x	y
0	-8
-1	-9
-2	-8
-4	0
2	0



Now let g(x) = 0 and solve for x.

$0 = x^2+2x-8$ $\quad x=-4 \longrightarrow$

$0=(x+4)(x-2)$ $\quad x=2 \longrightarrow$

What do these *x*-values correspond to on the graph? x intercepts

Conclusion: The "zeros" of a quadratic function are the same as the x int.

Recall to find the *y-intercept* set x = 0 and to find the *x-intercepts/zeros* set y = 0 and use the ZPP.

EX: $f(x) = x^2 + 7x - 60$

y int: $f(0) = 0^2+7(0)-60$

x int: $0=(x+12)(x-5)$

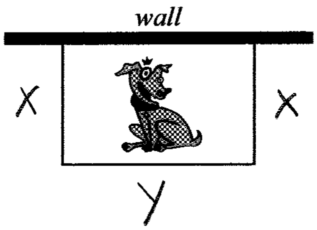
$x=-12$ $\quad x=5$

x	y
0	-60
-12	0
5	0

Word Problems Types:

1. Word problems that ask for the maximum or minimum. [Hint: find the Vertex]
2. Given a function $h(t)$ that describes the height of an object
   a) Find the max. height. [Hint: find the Vertex]
   b) Find when it hits the ground. [Hint: set $=0$]

1. Carlo plans to build a rectangular pen against an existing wall for his dog. He will buy 20 yards of fence material. Find the width and length needed to produce the maximum area. [Hint: find the Vertex .]



The vertex is (10 , 50). The max area occurs when $x =$ 5.

Then the width $y =$ 10

Dimensions: 5 by 10 Max Area: 50 yards^2

$X + y + X = 20$

$2x + y = 20$

$y = \boxed{-2x+20}$

$A = x \cdot y$

$A = x(-2x+20)$

$A = -2x^2 + 20x$

Vertex

$$X = \frac{-20}{2(-2)} = \frac{-20}{-4} = 5$$

$$A = -2(5)^2 + 20(5) = -2(25) + 100 = -50 + 100 = 50$$

2. A woman drops a front door key to her husband from their apartment window several stories about the ground. The function h(t) = -16t^2 +64 gives the height h of the key in feet, t seconds after she releases it. How long does it take for the key to reach the ground?

$0 = -16t^2 + 64$

$0 = -16(t^2 - 4)$

$0 = -16(t+2)(t-2)$

$t + 2 = 0$ $\boxed{t = 2}$

$\cancel{t = -2}$

negative time

The key will reach the ground after 2 seconds.

Date ________ Notes 4.6 Complete the Square
Square Root Theorem

Review: $\sqrt{81} = 9$ $\quad -\sqrt{144} = -12$ $\quad \pm\sqrt{4} = \pm 2$ $\quad \sqrt{25 \cdot 9} = 15$ $\quad \sqrt{12} = 2\sqrt{3}$ $\quad \sqrt{\frac{4}{9}} = \frac{2}{3}$

$\sqrt{25}\cdot\sqrt{9}$
$5 \cdot 3$
15

$\sqrt{4 \cdot 3}$
$\sqrt{4}\sqrt{3}$
$2\sqrt{3}$

$\frac{\sqrt{4}}{\sqrt{9}}$

Square Root Theorem: If $x^2 = c$, then $x = \pm\sqrt{c}$

Isolate the Square!

1. Solve: $2x^2 = 18$

$x^2 = 9$

$\sqrt{x^2} = \pm\sqrt{9}$

$\boxed{x = \pm 3}$

2. Solve: $7x^2 - \cancel{23} = 103 + 23$

$\frac{7x^2}{7} = \frac{126}{7}$

$x^2 = 18$

$\sqrt{x^2} = \pm\sqrt{18}$

$x = \pm\sqrt{9 \cdot 2}$ $\quad \boxed{x = \pm 3\sqrt{2}}$

3. Solve: $199 = 3(x+5)^2 + 7$

$\frac{192}{3} = \frac{3(x+5)^2}{3}$

$64 = (x+5)^2$

$\pm\sqrt{64} = \sqrt{(x+5)^2}$

$\pm 8 = x + 5$

$-5 \pm 8 = x$

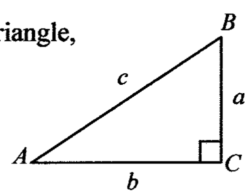
$-5 + 8 = x \quad -5 - 8 = x$

$\boxed{x = 3} \quad \boxed{x = -13}$

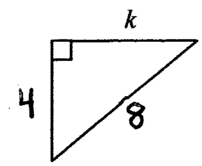
Pythagorean Theorem

If ΔABC is a right triangle, then

$a^2 + b^2 = c^2$



Find the unknown length in the right triangle. Round to nearest 10th.



$k^2 + 4^2 = 8^2$

$k^2 + 16 = 64$

$k^2 = 48$

$k = \pm\sqrt{48}$

$k = \pm\sqrt{16 \cdot 3}$

$k = \pm 4\sqrt{3}$

$\boxed{k = 4\sqrt{3}}$ Exact (only need positive)

$\boxed{k = 6.93}$ Approx.

Complete the Square

***Review*: Perfect Trinomial Squares fit a certain pattern.** $x^2 + 14x + 49 = (x+7)^2$

$x^2 - 22x + \underline{121} = (\underline{x} - \underline{11})^2$

This is called Completing the Square.
Take 1/2 of the coefficient (middle) of the x-term and Square it.

Examples – Complete the square and factor.

$x^2 - 6x + \underline{9} = (x-3)^2$

$\frac{1}{2}(-6) = (-3)^2 = 9$

$x^2 + 15x + \frac{225}{4} = (x + \frac{15}{2})^2$

$\frac{1}{2}(15) = \left(\frac{15}{2}\right)^2 = \frac{225}{4}$

$x^2 + bx + \frac{b^2}{4} = (x + \frac{b}{2})^2$

$\frac{1}{2}(b) = \left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$

Solve by completing the square.

$x^2+6x-16=0$

$x^2+6x=16$

$(x^2+6x+9)=16+9$

$(x+3)^2=25$

$\sqrt{(x+3)^2}=\pm\sqrt{25}$

$x+3=\pm5$

$x=-3+5=2$

$x=-3-5=-8$

$\boxed{x=2,-8}$

$x^2+10x-24=0$

$x^2+10x=24$

$x^2+10x+25=24+25$

$(x+5)^2=49$

$\sqrt{(x+5)^2}=\pm\sqrt{49}$

$x+5=\pm7$

$x=-5\pm7$

$x=-5+7=2$

$x=-5-7=-12$

$\boxed{x=2,-12}$

$2x^2=-6x+7$

$2x^2+6x=7$

$x^2+3x=\frac{7}{2}$

$\left(x^2+3x+\frac{9}{4}\right)=\frac{7}{2}+\frac{9}{4}$

$\left(x+\frac{3}{2}\right)^2=\frac{14}{4}+\frac{9}{4}$

$\left(x+\frac{3}{2}\right)^2=\frac{23}{4}$

$\sqrt{\left(x+\frac{3}{2}\right)^2}=\pm\sqrt{\frac{23}{4}}$

$x+\frac{3}{2}=\pm\frac{\sqrt{23}}{2}$

$\boxed{x=\frac{-3\pm\sqrt{23}}{2}}$

> Solve by completing the square:
> 1. Move the Constant to the other side.
> 2. Divide by the coefficient of x^2.
> 3. Complete the Square and add to other side.
> 4. Factor and solve by taking the Square Root of both sides. Don't forget $\pm$.

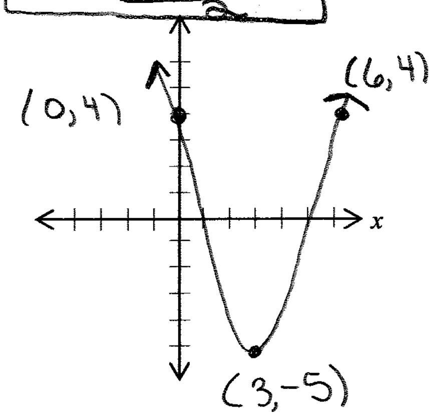
Connection to parabolas….

Graph $y=(x-3)^2-5$ State the transformations from the parent graph $y=x^2$.

$\rightarrow 3,\ \downarrow 5$

$y=x^2-6x+9-5$

$y=x^2-6x+4$



The form $y=(x-3)^2-5$ is called the Vertex form of a parabola because you can easily identify the Vertex.

How do you get a quadratic function in vertex form? Complete the Square!

Example: $y=x^2-12x+4$

$y\ \underline{-4}\ =x^2-12x$

$y-4+\underline{36}=(x^2-12x+\underline{36})$

$y+\underline{32}=(x-\underline{6})^2$

$y=\underline{(x-6)^2-32}$ Vertex: $\underline{(6,-32)}$

> **Vertex Form**
>
> A quadratic function, $y=ax^2+bx+c$ with $a\neq0$, can be written in vertex form by Completing the Square:
>
> $y=a(x-h)^2+k$ vertex (h,k)

Write the function in vertex form by completing the square. Give the coordinates of the vertex.

$y=x^2+4x+5$

$y-5=x^2+4x$

$y-5+4=x^2+4x+4$

$y-1=(x+2)^2$

$y=(x+2)^2+1$

$\boxed{V(-2,1)}$

$y=2x^2+16x+13$ (careful!)

$y-13=2x^2+16x$

$y-13=2(x^2+8x\quad)$

$y-13+32=2(x^2+8x+16)$

$y+19=2(x+4)^2$

$y=2(x+4)^2-19$

$\boxed{V=-4,-19}$

Putting it all together… **Given the function** $f(x) = x^2 - 2x - 3$

Sketch. Label key points and axis.

a) Opens up and has a minimum

b) y-intercept: $(0, -3)$

c) Zeros (set $f(x) = 0$)

$0 = x^2 - 2x - 3$

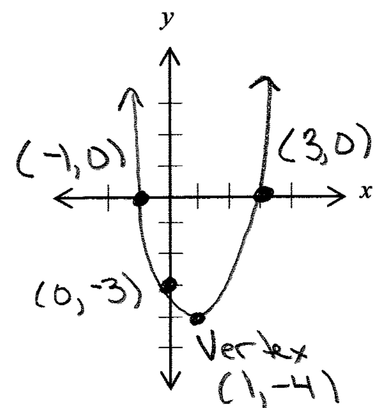
$0 = (x-3)(x+1)$

$x - 3 = 0 \qquad x + 1 = 0$

$x = 3 \qquad x = -1$

d) Vertex: $(1, -4)$

Minimum : -4



e) Axis of symmetry: $x = 1$

f) State transformations.

Right 1, Down 4

Vertex

Method #1

$x = \frac{-b}{2a}$

$x = \frac{-(-2)}{2(1)}$

$x = 1$

$f(1) = 1^2 - 2(1) - 3$

$y = 1 - 2 - 3$

$y = -4$

Vertex $(1, -4)$

Method #2

$y = x^2 - 2x - 3$

$y + 3 = x^2 - 2x$

$y + 3 + 1 = x^2 - 2x + 1$

$y + 4 = (x-1)^2$

$y = (x-1)^2 - 4$

Date ________________ Notes 4.7: Quadratic Formula

Quadratic Formula

If $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Follow the same steps to "complete the square."

$$ax^2 + bx + c = 0$$

$\frac{1}{2} \cdot \frac{b}{a} = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

Proof:

$ax^2 + bx = -c$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{-c \cdot 4a}{a \cdot 4a} + \frac{b^2}{4a^2}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$

$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

$\boxed{x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}}$

Use the quadratic formula to solve. Give exact solutions.

$3x^2 = 4x + 6$

$3x^2 - 4x - 6 = 0$ $\quad a = 3,\ b = -4,\ c = -6$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-6)}}{2(3)}$

$x = \frac{4 \pm \sqrt{16 + 72}}{6}$

$x = \frac{4 \pm \sqrt{88}}{6}$

$x = \frac{4 \pm \sqrt{4 \cdot 22}}{6}$

$x = \frac{4 \pm 2\sqrt{22}}{6}$

$x = \frac{2(2 \pm \sqrt{22})}{6}$

$\boxed{x = \frac{2 \pm \sqrt{22}}{3}}$ Exact

Your school's jazz band is selling CDs as a fundraiser. The total profit **y** depends on the amount x that your band charges for each CD. The equation:

$$y = -x^2 + 48x - 300$$

models the profit of the fundraiser. What is the least amount in dollars you can charge for a CD to make a profit of $200?

$200 = -x^2 + 48x - 300$

$x^2 - 48x + 500 = 0$

$x = \frac{48 \pm \sqrt{48^2 - 4(1)(500)}}{2(1)}$

$x = \frac{48 \pm \sqrt{304}}{2}$

$x = 32.72$ $\quad \boxed{x = 15.28}$ least

The least amount you can charge is \$15.28

Find the zeros/x-intercepts/roots by setting $y = 0$ and using the Quadratic Formula.

1. $y = x^2 + 12x + 35$

$0 = x^2 + 12x + 35$

$x = \frac{-12 \pm \sqrt{144 - 4(1)(35)}}{2(1)}$

$x = \frac{-12 \pm \sqrt{4}}{2}$

$x = \frac{-12 \pm 2}{2}$

$x = -10/2$ $\quad x = -14/2$

$\boxed{x = -5}$ $\quad \boxed{x = -7}$

$\boxed{\text{x int } (-5,0)\ (-7,0)}$

\# of real roots: 2

\# of x-intercepts: 2

2. $y = x^2 - 6x + 9$

$0 = x^2 - 6x + 9$

$x = \frac{6 \pm \sqrt{36 - 4(1)(9)}}{2(1)}$

$x = \frac{6 \pm \sqrt{36 - 36}}{2}$

$x = \frac{6 \pm \sqrt{0}}{2}$

$\boxed{x = 3}$ $\quad \boxed{\text{x int } (3,0)}$

\# of real roots: 1

\# of x-intercepts: 1

3. $y = x^2 + 4x + 9$

$0 = x^2 + 4x + 9$

$x = \frac{-4 \pm \sqrt{16 - 4(1)(9)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{16 - 36}}{2}$

$x = \frac{-4 \pm \sqrt{-20}}{2}$

$\boxed{\text{No Real Solutions}}$

\# of real roots: 0

\# of x-intercepts: 0

The expression under the radical, $b^2 - 4ac$ is called the Discriminant

*Discriminant = 4 $\qquad$ *Discriminant = 0 $\qquad$ *Discriminant = -20

In the quadratic equation $ax^2 + bx + c = 0$, the **Discriminant** = $b^2 - 4ac$

- If $b^2 - 4ac > 0$, then the equation has 2 distinct real solutions. There are 2 x-intercepts.
- If $b^2 - 4ac = 0$, then the equation has 1 real solution, a double root. There is 1 x-intercept.
- If $b^2 - 4ac < 0$, then the equation has 0 real solutions. There are NO x-intercepts.

*Note: If the discriminant is a perfect square, the equation is factorable!

Find the discriminant and determine the number of *real* solutions without solving the equation.

1. $3x^2 - 6x + 4 = 0$

$b^2 - 4ac$

$(-6)^2 - 4(3)(4)$

$36 - 48$

$\boxed{-12}$

$\boxed{\text{No Real Sol.}}$

2. $3x^2 = 6x - 3$

$3x^2 - 6x + 3 = 0$

$b^2 - 4ac$

$(-6)^2 - 4(3)(3)$

$36 - 36$

$\boxed{0}$

$\boxed{\text{1 real solution, double root}}$

3. $3x^2 - 2x + 7 = 4x + 5$

$3x^2 - 6x + 2 = 0$

$b^2 - 4ac$

$(-6)^2 - 4(3)(2)$

$36 - 24$

$\boxed{12}$

$\boxed{\text{2 Real Solutions}}$

Date:____________

Notes 4.8: Complex Numbers

To solve negatives under a square root, imaginary **numbers were developed, where** $i = \sqrt{-1}$ or $i^2 = -1$.

Simplify: $\sqrt{-9} = \sqrt{(-1)(9)} = \boxed{3i}$ $\sqrt{-32} = \sqrt{(-1)(16)(2)} = \boxed{4i\sqrt{2}}$ $\sqrt{-9} \cdot \sqrt{-16} =$

Convert to imaginary numbers before multiplying

$\sqrt{(-1)(9)} \cdot \sqrt{(-1)(16)}$

$3i \cdot 4 \cdot i = 12i^2 =$

$12(-1) = \boxed{-12}$

Complex Number

Form: $a+bi$

where a and b are real numbers.

$i = \sqrt{-1}$ or $i^2 = -1$.

a is the Real part; b is the imaginary part.

Powers of i:

$i^1 = \boxed{i}$

$i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1}$ $(\sqrt{-1})^2 = \boxed{-1}$

$i^3 = i \cdot i^2 = i(-1) = \boxed{-i}$

$i^4 = i^2 \cdot i^2 = (-1)(-1) = \boxed{1}$

$i^5 = i^4 \cdot i^1 = i$

$i^6 = i^4 \cdot i^2 = -1$

$i^7 = i^4 \cdot i^3 = -i$

$i^8 = i^4 \cdot i^4 = 1$

Conclusion:

The powers of i are cyclic and repeat in a pattern of four numbers: i, -1, $-i$, and 1.

R1 R2 R3 R0

Ex: $i^{14} = i^4 \cdot i^4 \cdot i^4 \cdot i^2$

$= 1 \cdot 1 \cdot 1 \cdot i^2 = \boxed{-1}$

Try... $i^{35} = i^3 = \boxed{-i}$ $i^{57} = i^1 = \boxed{i}$ $i^{96} = \boxed{1}$

$35 \div 4 = 8$, 32, $3 \leftarrow i^3$ "i^4 8 times 3 remaining"

$57 \div 4 = 14$, 4, 17, 16, 1 $i^1 \rightarrow 1$

$96 \div 4 = 24$, 8, 16 R0

Operations with Complex Numbers:

1. $(-10-6i)+(8-4i)$

$\boxed{-10} - 6i \boxed{+8} - 4i$

$\boxed{-2-10i}$

2. $(-10-6i)-(8-4i)$

$\boxed{-10} - 6i \boxed{-8} + 4i$

$\boxed{-18-2i}$

3. $(2-6i)(3-4i)$

$6 \boxed{-8i} \boxed{-18i} + 24i^2$

$6 - 26i + 24(-1)$

$6 - 26i - 24$

$\boxed{-18-26i}$

4. $(6-4i)(5+2i)$

$30+12i-20i-8i$

$30-8i-8(-1)$

$30-8i+8$

$\boxed{38-8i}$

5. $(5-3i)(5+3i)$

$25+15i-15i-9i^2$

$25-9i^2 = 25+9 = \boxed{34}$

6. $(3+4i)(3-4i)$

$9-12i+12i-16i^2$

$9-16i^2 = 9+16 = \boxed{25}$

The Conjugate of $a+bi$ is $a-bi$. $(a+bi)(a-bi) =$

Dividing Complex Numbers: Multiply the numerator **and the** denominator **by the** Conjugate **of the** denominator. **We do this because** $i = \sqrt{-1}$ **and** $\sqrt{\ }$ **aren't simplified if in the denominator.**

7. $\dfrac{2+5i}{2-3i} \cdot \dfrac{2+3i}{2+3i}$

$\dfrac{4+6i+10i+15i^2}{4-9i^2}$

$\dfrac{4+16i-15}{4+9} = \boxed{\dfrac{-11+16i}{13}} = \boxed{\dfrac{-11}{13} + \dfrac{16i}{13}}$

8. $\dfrac{3-4i}{5+2i} \cdot \dfrac{5-2i}{5-2i}$

$\dfrac{15-6i-20i+8i^2}{25-4i^2}$

$\dfrac{15-26i-8}{25+4} = \boxed{\dfrac{7-26i}{29}} = \boxed{\dfrac{7}{29} - \dfrac{26}{29}}$

Now, we can solve problems that have no real solutions!

Solve using the quadratic formula over the complex numbers:

1) $x^2 - 4x + 5 = 0$

$$x = \frac{4 \pm \sqrt{16-4(1)(5)}}{2(1)} \qquad x = \frac{4 \pm \sqrt{(-1)(4)}}{2}$$

$$x = \frac{4 \pm \sqrt{16-20}}{2} \qquad x = \frac{4 \pm 2i}{2}$$

$$x = \frac{4 \pm \sqrt{-4}}{2} \qquad x = \frac{\cancel{2}(2 \pm i)}{\cancel{2}}$$

$$\boxed{x = 2 \pm i}$$

2) $4x^2 = 2x - 1$ $\quad 4x^2 - 2x + 1 = 0$

$$x = \frac{2 \pm \sqrt{4-4(4)(1)}}{2(4)}$$

$$x = \frac{2 \pm \sqrt{4-16}}{8} \qquad x = \frac{2 \pm 2i\sqrt{3}}{8}$$

$$x = \frac{2 \pm \sqrt{-12}}{8} \qquad x = \frac{2(1 \pm i\sqrt{3})}{8}$$

$$x = \frac{2 \pm \sqrt{-4 \cdot 3}}{8} \qquad x = \frac{1 \pm i\sqrt{3}}{4}$$

$$\boxed{x = \frac{1}{4} \pm \frac{i\sqrt{3}}{4}}$$

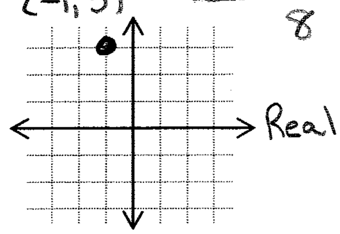
Extensions:

Graph a complex number.

Label the x-axis the Real axis and the y-axis the imaginary axis.

To graph $-1+3i$, plot the point $(-1, 3)$.

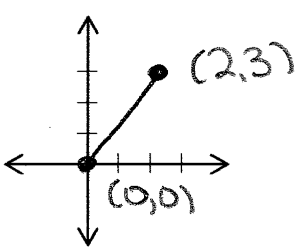
Compare to x Compare to y



The absolute value or Modulus **of a complex number is the** distance **from the** origin.

Graph and find the absolute value (modulus)

1. $|2+3i|$



$$d = \sqrt{(2-0)^2 + (3-0)^2}$$

$$d = \sqrt{4+9}$$

$$\boxed{d = \sqrt{13}}$$

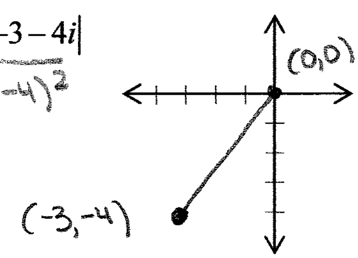
2. $|-3-4i|$

$$d = \sqrt{(-3)^2 + (-4)^2}$$

$$d = \sqrt{9+16}$$

$$d = \sqrt{25}$$

$$\boxed{d = 5}$$



Absolute Value (or Modulus) of a Complex Number

$$|a+bi| = \sqrt{a^2+b^2}$$

Graphically, $|a+bi|$ represents the distance from the origin.

Date ______________ Notes 4.9: Quadratic Systems

> To solve a one-variable quadratic inequality by **region testing**:
>
> 1. Factor and find the Critical #s. ✫ Set equal to zero and solve
> 2. Graph the Critical #s on a number line (use open or Closed circles)
> 3. Test a number from each region into the original inequality to determine yes or no for each factor.
> 4. Decide if you want the ____________ or ____________ solutions. "does it work?"

Problems – Solve by *region testing*.

$x^2 - 8x + 12 \geq 0$

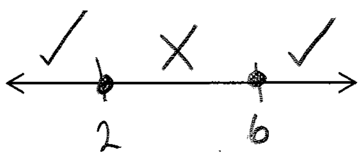
closed

Critical Values

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 6, 2$



Test $x=0$: $0^2 - 8(0) + 12 \geq 0$ $\quad 12 \geq 0$ ✓

Test $x=5$: $25 - 40 + 12 \geq 0$ $\quad -13 \geq 0$ X

Test $x=7$: $49 - 56 + 12 \geq 0$ $\quad 5 \geq 0$ ✓

Solution: $\{x \mid x \leq 2 \text{ or } x \geq 6\}$

closed

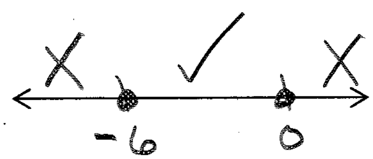
$x^2 \leq -6x$

Critical Values

$x^2 + 6x = 0$

$x(x+6) = 0$

$x = 0, x = -6$



Test $x=-7$: $(-7)^2 \leq (-6)(-7)$ $\quad 49 \leq 42$ no

Test $x=-1$: $(-1)^2 \leq (-6)(-1)$ $\quad 1 \leq 6$ yes

Test $x=1$: $(1)^2 \leq (-6)(1)$ $\quad 1 \leq -6$ no

Solution: $\{x \mid -6 \leq x \leq 0\}$

open

$3x^2 < x + 2$

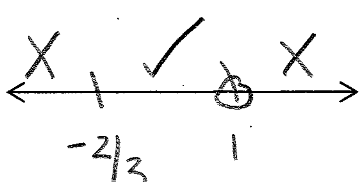
Critical Values

$3x^2 - x - 2 = 0$

$(3x+2)(x-1) = 0$

$3x + 2 = 0 \quad x - 1 = 0$

$x = -2/3 \quad x = 1$



Test $x=-1$ $\quad 3(-1)^2 < -1 + 2$ $\quad 3 < 1$ NO

Test $x=0$ $\quad 3(0)^2 < 0 + 2$ $\quad 0 < 2$ Yes

Test $x=2$ $\quad 3(2)^2 < 2 + 2$ $\quad 12 < 4$ NO

Solution: $\{x \mid -2/3 < x < 1\}$

open

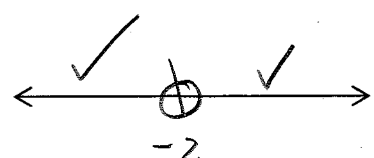
$x^2 + 4x > -4$

Critical Values

$x^2 + 4x + 4 = 0$

$(x+2)(x+2) = 0$

$x = -2$



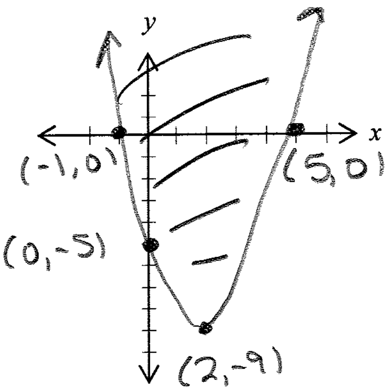
Test $x=-3$: $(-3)^2 - 12 > -4$ $\quad -3 > -4$ yes

Test $x=0$: $0^2 + 4(0) > -4$ $\quad 0 > -4$

Solution: $\{x \mid x < -2 \text{ or } x > -2\}$

Two Variable Quadratic Inequalities

Sketch the graph of $y \geq x^2 - 4x - 5$. Solid Line



1. Graph $y = x^2 - 4x - 5$ by finding the

<u>y-intercept</u>: $(0,-5)$

<u>zeros</u>: $0 = (x-5)(x+1)$, $x = 5, -1$, $(5,0)\ (-1,0)$

<u>vertex</u>: $x = \frac{4}{2(1)} = 2$, $y = 2^2 - 4(2) - 5$, $y = 4 - 8 - 5$, $y = -9$, V $(2,-9)$

2. Draw a Solid curve.

3. Test $(0,0)$ to shade inside or outside the parabola.

$0 \geq 0^2 - 4(0) - 5$

$0 \geq -5$ yes

* into original

2. Sketch the graph of $y > -(x-1)^2 + 4$.

Graph $y = -(x-1)^2 + 4$

<u>Vertex</u>: $(1, 4)$

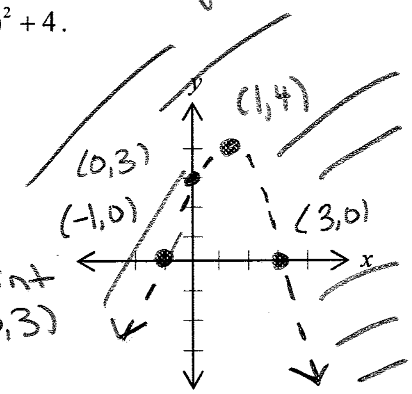
<u>y-intercept</u>: set $x = 0$

$y = -(x^2 - 2x + 1) + 4$

$y = -x^2 + 2x - 1 + 4$

$y = -x^2 + 2x + 3$

yint $(0,3)$



<u>Zeros</u>: set $y = 0$

$0 = -x^2 + 2x + 3$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = 3, -1$

$(3,0)(-1,0)$

Test $(0, 0)$:

$0 > -(0-1)^2 + 4$

$0 > 1 + 4$

$0 > 5$ NO

Summarize how to graph a **quadratic inequality involving <u>two</u> variables.**

1. Graph the parabola using Vertex, Zeros, and y - intercept.
2. Solid vs. Dotted curve
3. Test a point to shade inside or outside the parabola.