Date _______________ 5.1 Notes Polynomial Functions

Definitions:

Monomial – a number, a variable, or a product of a number and variables.

Examples: $5, x, 5x^2, \ldots$

Polynomial – a monomial or the sum / difference of monomial terms

1 term = monomial
2 terms = binomial
3 terms = trinomial
$\geq$ 4 terms = polynomial

Definition of a Polynomial Function is of the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

→ "a": real #'s
"n" = non-neg integers

Degree of polynomial = exponent of the highest power of x.

Ex: $4x^2 - 3x + 2x^7 - 8 \rightarrow$ degree = 7

The **leading coefficient** is the non-zero coeff. of the highest power of x.

Special polynomial functions:

- If the degree is 1, the function is linear.
- If the degree is 2, the function is quadratic.
- If the degree is 3, the function is cubic.
- If the degree is 4, the function is quartic.

Example: Write each polynomial in standard form. Then classify by the degree and number of terms.

1. $3x + 9x^2 + 5$

$\boxed{9x^2 + 3x + 5}$

quadratic trinomial

2. $4x - 6x^2 + x^4 + 10x^2 - 12$

$\boxed{x^4 + 4x^2 + 4x - 12}$

quartic polynomial

End behavior of a polynomial function.

If the degree is **even**, then the ends of the graph go in the same direction.

If the degree is **odd**, then the ends of the graph go in opposite directions.

The end behavior of a polynomial function describes what the graph looks like

as $x \rightarrow -\infty$ (x gets very small / left end)

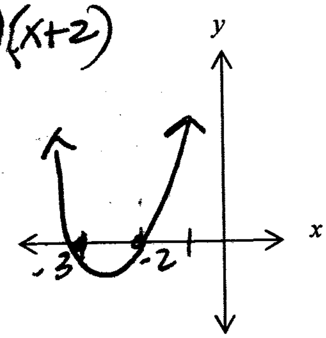
as $x \rightarrow \infty$ (x gets very big / right end)

Hint: look at the leading term.

Examples:

$f(x) = x^2 + 5x + 6 \Rightarrow$

Check with a sketch. $(x+3)(x+2)$



$(-\infty)$ **Left End of graph:**

as $x \to$ $\underline{-\infty}$, $y \to$ $\underline{\infty}$

as x gets very small, y gets very big

In words: on the left, the graph rises

$(+\infty)$ **Right End of graph:**

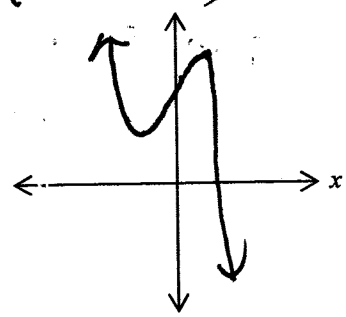
as $x \to$ $\underline{\infty}$, $y \to$ $\underline{\infty}$

as x gets very big, y gets very big

In words: on the right, the graph rises

$f(x) = -2x^3 + 5x^2 + x + 2$

Check with a sketch. (use TI)



Left End: $-2(-\infty)^3 \to +\infty$

as $x \to$ $\underline{-\infty}$, $y \to$ $\underline{\infty}$

as x gets very small, y gets very big

In words: on the left, the graph rises

Right End:

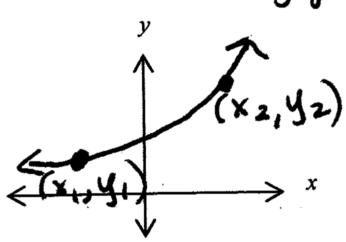
as $x \to$ $\underline{\infty}$, $y \to$ $\underline{-\infty}$

as x gets very big, y gets very small

In words: on the right, the graph falls

Increasing Function

as x gets bigger, y gets bigger

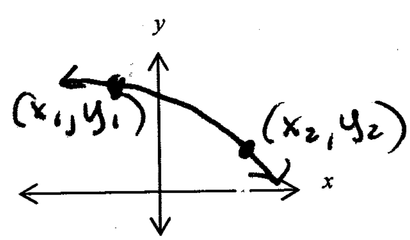


Given two points (x_1, y_1) and (x_2, y_2),

if $x_2 > x_1$, then $y_2 > y_1$

Decreasing Function

as x gets bigger, y gets smaller



Given two points (x_1, y_1) and (x_2, y_2),

if $x_2 > x_1$, then $y_2 < y_1$

Polynomial functions are often increasing for certain interval(s) and decreasing for other interval(s).

****Note: use the X-values for incr/decr intervals.***

Example: Graph using your TI.

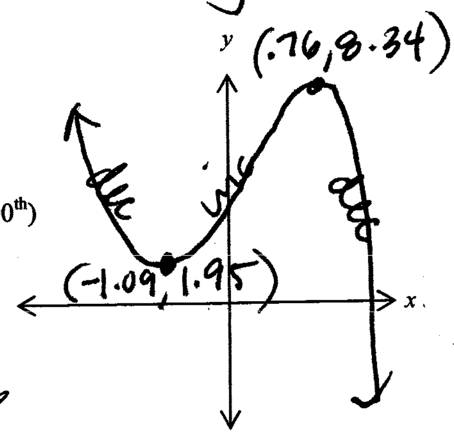
$f(x) = -2x^3 - x^2 + 5x + 6$

Relative (or Local) Max. = $\underline{8.34}$ when $x = \underline{0.76}$ (nearest 100[th])

Relative (or Local) Min. = $\underline{1.95}$ when $x = \underline{-1.09}$

Increasing Interval(s): $-1.09 \le x \le 0.76$

Decreasing Interval(s): $x \le -1.09$ or $x \ge 0.76$



Relative (Local) Extrema (Maxima or Minima):

A polynomial function of degree n has at most n zeros.

A polynomial function of degree n has at most $n-1$ relative extrema or turning points.

A polynomial function of degree n has at most $n-2$ points of inflection. (A point of inflection is where the graph has a change in concavity.)

The following is a complete graph of a polynomial function f.

Is the degree of f even or odd? even (same directions)

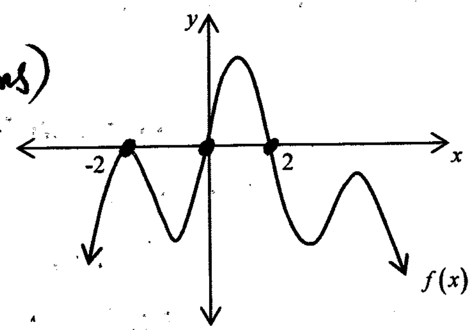
Is the leading coefficient of f positive or negative? neg (down)

What are the real zeros of f? $-2, 0, 2$

What is the smallest possible degree of f?

5 turning points, 3 zeros

degree → $\boxed{n=6}$



Conclusions: Given a polynomial function of degree n.

1. The **# of U turns** = at most $n-1$
2. The **Domain** of a polynomial function = $\mathbb{R}$
3. **End Behavior** of a polynomial function
   a) degree is **odd**: ends go in opposite directions
   b) degree is **even**: ends go in same directions
4. The **Range** of a polynomial function
   a) degree is **odd**, R = $\mathbb{R}$
   b) degree is **even**, R = $y \leq c$ or $y \geq c$ *where "c" is the max/min value.

Date ________________ 5.2 Notes Polynomials, linear Factors, zeros

The following are equivalent statements about a real number b and a polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$:

- $x - b$ is a *linear factor of the polynomial* $P(x)$.
- b *is a* zero *of the polynomial function* $y = P(x)$.
- b *is a* root *(or solution) of the polynomial equation* $P(x) = 0$.
- b *is an* x-interupt *of the graph* $y = P(x)$.

Example. Given the function $y = (x+2)(x-1)(x-3)$:

a) What are the linear factors? $(x+2), (x-1), (x-3)$

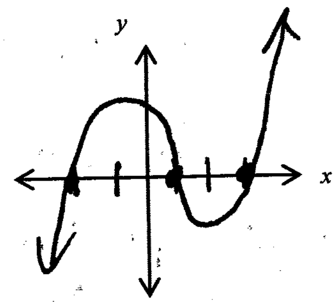
b) What are the zeros? $-2, 1, 3$

c) What are the roots/solutions? $-2, 1, 3$

d) What are the x-intercepts? $(-2,0)(1,0)(3,0)$

e) Graph the function. (Think: end behavior?)

leading term $\rightarrow x^3$



Factor Theorem: $x - b$ is a factor of the polynomial $P(x) \Leftrightarrow$ $P(b)=0$

1. Find a cubic polynomial with zeros 1, -3, 5

$y = (x-1)(x+3)(x-5)$

$y = (x-1)(x^2 - 2x - 15)$

$y = x^3 - 2x^2 - 15x - x^2 + 2x + 15$

$\boxed{y = x^3 - 3x^2 - 13x + 15}$

2. Find a quartic polynomial with zeros 1, 1, 3 and −3.

$y = (x-1)^2(x-3)(x+3)$

$y = (x^2 - 2x + 1)(x^2 - 9)$

$y = x^4 - 2x^3 + x^2 - 9x^2 + 18x - 9$

$\boxed{y = x^4 - 2x^3 - 8x^2 + 18x - 9}$

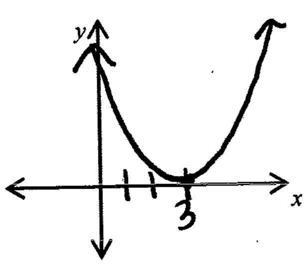
Is there more than one answer? Explain.

Yes! there should be "a" out in front. Not enough info to find a in these equations.

Multiplicity: **The number of times the same number is a** zero.

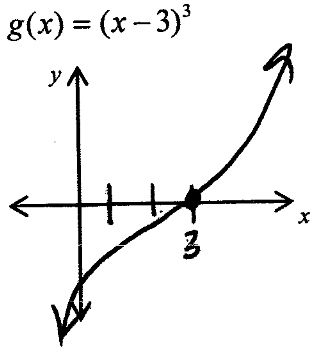
Compare the graphs using your TI:

$f(x) = (x-3)^2$



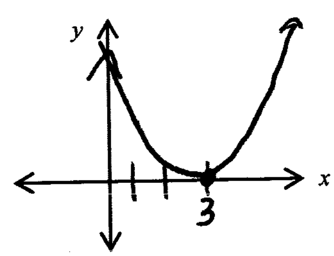
$x =$ 3 is a double root (touches)

$g(x) = (x-3)^3$



$x =$ 3 is a triple root (crosses)

$h(x) = (x-3)^4$



$x =$ 3 is a quadruple root (touches)

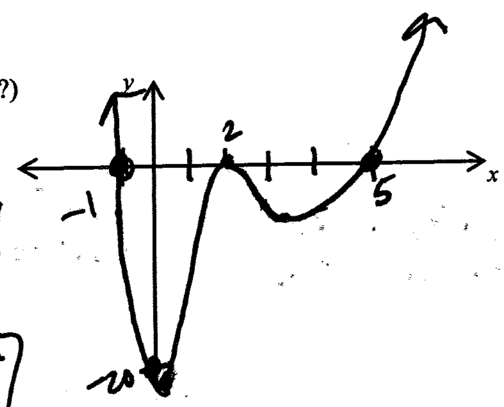
Summarize your conclusions: $f(x)=(x-c)^k \Rightarrow$ c is a zero of multiplicity K.

1. If a root has odd multiplicity, then the graph crosses the x-axis.
2. If a root has even multiplicity, then the graph touches the x-axis. (tangent)

Analyze the function without graphing. $f(x)=(x-5)(x-2)^2(x+1)^3$

Identify and analyze the zeros? (Do they cross the x-axis or just touch it?)

X=5 crosses   x=2 touches   x=-1 crosses



What is the y-intercept?

$X=0 \rightarrow (-5)(-2)^2(1)^3=-20$ $\boxed{(0,-20)}$

What is the end behavior?

$\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = \infty$

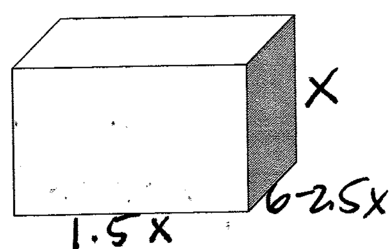
$f(x)=(x^1)(x^2)(x^3)=x^6+\ldots$ [even & pos]

Make a sketch based on your predictions. Check with your TI.

Using a polynomial function to maximize volume.

The design of a digital box camera maximizes the volume while keeping the sum of the dimensions at six inches. If the length must be 1.5 times the height, what should each dimension be?

Let x = height of the camera



Find expressions for the length and width.

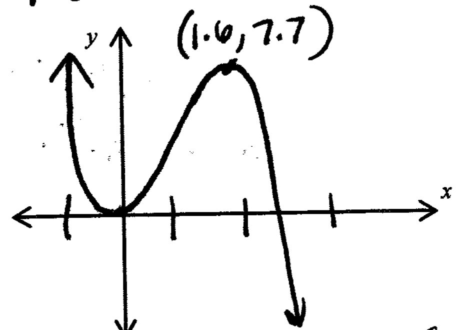
$\boxed{L=1.5X}$ $W=6-(X+1.5X)=\boxed{6-2.5X}$

Write the volume as a function of x: L·W·H

$V(x)= x(1.5x)(6-2.5x)$

$\boxed{V(x)=-3.75x^3+9x^2}$

Graph the function. Calculate the maximum. What does each value tell you?



max = 7.7 when x = 1.6.
(volume) (height)

Write your solution to this problem in a complete sentence.

In order to get a maximum volume of 7.7 in³, the height would be 1.6 in, the width would be 2 in and the length would be 2.4 in.

$W=6-2.5(1.6)$
W = 2 in
$L=1.5(1.6)$
L = 2.4 in

Date ______________ 5.3 Notes Solving Poly. Eqns

Review:

Factor Patterns:

Difference of Two Squares: $a^2 - b^2 = (a-b)(a+b)$

Perfect Trinomial Squares: $a^2 - 2ab + b^2 = (a-b)^2$ $a^2 + 2ab + b^2 = (a+b)^2$

Multiplying polynomials. Write your answer in standard form.

$x(x-1)(x+4)$

$x(x^2+3x-4)$

$\boxed{x^3+3x^2-4x}$

$(x+1)(x-1)(x-3)$

$(x^2-1)(x-3)$

$\boxed{x^3-3x^2-x+3}$

$(x+1)(x-2)(x+3)$

$(x+1)(x^2+x-6)$

$x^3+x^2-6x+x^2+x-6$

$\boxed{x^3+2x^2-5x-6}$

Solve by factoring and applying the ZPP over the complex numbers.

$2x^3 - 32x^2 + 120x = 0$

$2x(x^2-16x+60)=0$

$2x(x-6)(x-10)=0$

$\boxed{x=0,6,10}$

$x^3 + 6x^2 - 5x - 30 = 0$

$x^2(x+6)-5(x+6)=0$

$(x^2-5)(x+6)=0$

$\boxed{x=\pm\sqrt{5},-6}$

$2x^2 + 24x + 72 = 0$

$x^2+12x+36=0$

$(x+6)^2=0$

$\boxed{x=-6}$

$81x^4 - 16 = 0$

$(9x^2-4)(9x^2+4)=0$

$(3x-2)(3x+2)(9x^2+4)=0$

$\boxed{x=\pm 2/3}$

$9x^2=-4$

$\sqrt{x^2}=\sqrt{\frac{-4}{9}}$

$\boxed{x=\pm\frac{2i}{3}}$

Sum of Two Cubes: $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

Difference of Two Cubes: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

memorize!! [S.O.A.P.]

Examples:

a) $x^3 - 1000 = (\underline{x})^3 - (\underline{10})^3$

$= \boxed{(x-10)(x^2+10x+100)}$

b) $x^3 + 27 = (x)^3+(3)^3$

$= \boxed{(x+3)(x^2-3x+9)}$

c) $8x^3 - 1 = (2x)^3-1^3$

$= \boxed{(2x-1)(4x^2+2x+1)}$

d) $125x^3 + 64 = (5x)^3+(4)^3$

$= \boxed{(5x+4)(25x^2-20x+16)}$

Solve algebraically.

a) $x\left[\frac{1}{x}+2\right]=[3x]x$

$1+2x=3x^2$

$0=3x^2-2x-1$

$0=(3x+1)(x-1)$

$\boxed{x=-1/3,1}$

b) $x^3 + 2x^2 = 9x + 18$

$x^3+2x^2-9x-18=0$

$x^2(x+2)-9(x+2)=0$

$(x^2-9)(x+2)=0$

$(x-3)(x+3)(x+2)=0$

$\boxed{x=\pm 3,-2}$

c) $x^3 + 5 = 4x^2 + x$ … What can you do if you can't factor and solve?

$x^3-4x^2-x+5=0$

. . . ?

Solving equations graphically: 2 different methods

Example (c): $x^3 + 5 = 4x^2 + x$. → $x^3 + 5 - 4x^2 - x = 0$

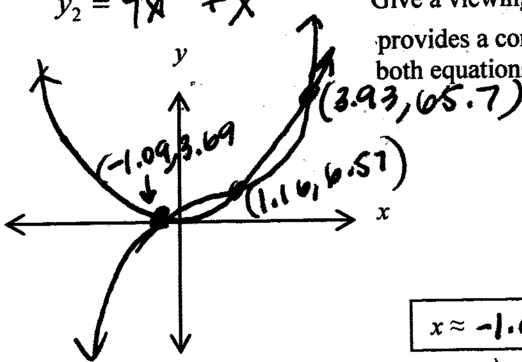
Method 1: intersection Method

Let $y_1 = x^3 + 5$

$y_2 = 4x^2 + x$

Use color and label each one.

Give a viewing window that provides a complete graph for both equations. Label key points.



$x \approx -1.09, 1.16, \boxed{3.93}$

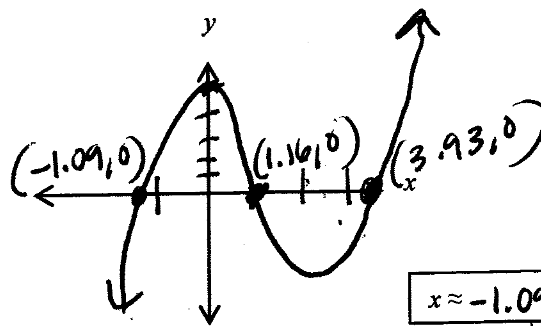
* x-values only!

[-5, 5] by [-5, 80]

Xmin, Xmax Ymin, Ymax

Method 2: zero or x-int Method

Let $y_1 = x^3 - 4x^2 - x + 5$



$x \approx -1.09, 1.16, 3.93$

[-10, 10] by [-10, 10]

The solutions are your x **-values only. Why?** You are only solving for x in the original equation.

In general, solve $h(x) = g(x)$ by graphing:

Method 1: intersection Method

1. Graph $y_1 =$ h(x) and $y_2 =$ g(x).
2. Find each point of intersection.
3. The solutions are the x-values of each point of intersection.

Method 2: zero or x-int Method

1. Rewrite the equation as h(x) − g(x) = 0.
2. Graph $y_1 =$ h(x) − g(x)
3. Find the x-intercepts or zeros.
4. The solutions are the x-values.

Date ______________ 5.4 Notes Dividing Polynomials

Review:

Division Vocabulary

Example: $28 \div 7 = 4$ → This example is written as a quotient statement.

The *dividend* is 28

Rewrite as a "product" statement: $(7)(4) = 28$

The *divisor* is 7

Rewrite as a "long division" problem:

$$\begin{array}{r} 4 \\ 7\overline{)28} \\ -28 \\ \hline 0 \end{array}$$

The *quotient* is 4

Long division with numbers: quotient

Quotient Statement: or Product Statement:

$$684 \div 16 \Rightarrow \begin{array}{r} 42 \\ 16\overline{)684} \\ 64 \\ \hline 44 \\ -32 \\ \hline 12 \end{array}$$

divisor

remainder

$684 \div 16 = 42 + \frac{12}{16}$

$\left[q + \frac{r}{d}\right]$

$684 = 42(16) + 12$

$[(q)(d) + r]$

Long division with polynomial functions:

1. Find the quotient and the remainder: $\frac{x^3 - 6x^2 + 10x - 8}{x - 4} =$

$$\begin{array}{r} x^2 - 2x + 2 \\ x-4\overline{)x^3 - 6x^2 + 10x - 8} \\ -(x^3 - 4x^2) \qquad\qquad \\ \hline -2x^2 + 10x \qquad \\ -(-2x^2 + 8x) \qquad \\ \hline 2x - 8 \\ -(2x - 8) \\ \hline \boxed{0} \end{array}$$

Write as a quotient statement: $(x^3 - 6x^2 + 10x - 8) \div (x - 4) =$ $x^2 - 2x + 2$

Write an equivalent "product" statement: $x^3 - 6x^2 + 10x - 8 =$ $(x - 4)(x^2 - 2x + 2)$

$\Rightarrow$ $x - 4$ is a factor of $x^3 - 6x^2 + 10x - 8$.

2. Find the quotient and the remainder: $(3x^4 - 8x^2 + 11x + 1) \div (x+2) =$

$$
\begin{array}{r|l}
 & 3x^3 - 6x^2 + 4x + 3 \\
x+2 & 3x^4 + 0x^3 - 8x^2 + 11x + 1 \\
 & -(3x^4 + 6x^3) \\
 & -6x^3 - 8x^2 \\
 & -(-6x^3 - 12x^2) \\
 & 4x^2 + 11x \\
 & -(4x^2 + 8x) \\
 & 3x + 1 \\
 & -(3x + 6) \\
 & -5
\end{array}
$$

Write as a quotient statement: $(3x^4 - 8x^2 + 11x + 1) \div (x+2) =$ $3x^3 - 6x^2 + 4x + 3 - \frac{5}{x+2}$

Write an equivalent "product" statement: $3x^4 - 8x^2 + 11x + 1 =$ $(x+2)(3x^3 - 6x^2 + 4x + 3) - 5$

$\Rightarrow$ $x+2$ is NOT a factor of $3x^4 - 8x^2 + 11x + 1$.

Division Algorithm:
If a polynomial $P(x)$ is divided by a nonzero polynomial $D(x)$, then there is a quotient polynomial $Q(x)$ and a remainder polynomial $R(x)$ such that $P(x) =$ $D(x) \cdot Q(x) + R(x)$

3. Determine if $x^2 + 9$ is a factor of $x^5 + x^4 - 81x - 81$. Write a product statement for your final answer.

$$
\begin{array}{r|l}
 & x^3 + x^2 - 9x - 9 \\
x^2+9 & x^5 + x^4 + 0x^3 + 0x^2 - 81x - 81 \\
 & -(x^5 + 9x^3) \\
 & x^4 - 9x^3 + 0x^2 \\
 & -(x^4 + 9x^2) \\
 & -9x^3 - 9x^2 - 81x \\
 & -(-9x^3 - 81x) \\
 & -9x^2 - 81 \\
 & -(-9x^2 - 81) \\
 & 0
\end{array}
$$

Final answer: $x^5 + x^4 - 81x - 81 =$ $(x^2+9)(x^3 + x^2 - 9x - 9)$

$r = 0 \therefore x^2 + 9$ is a factor!

Factor Theorem:
A polynomial function $P(x)$ has a linear factor $x - c$ $\Leftrightarrow P(c) =$ 0

Synthetic division is a shortcut to long division This shortcut only works on division with linear factors.

$(x^3 - x^2 - 17x - 15) \div (x+3)$

$$\begin{array}{r|rrrr} -3 & 1 & -1 & -17 & -15 \\ & & -3 & 12 & 15 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

Coefficients

$(x^3 - x^2 - 17x - 15) \div (x+3) =$ x^2-4x-5 or

$x^3 - x^2 - 17x - 15 =$ $(x+3)(x^2-4x-5)$

Try: $(x^3 - x^2 - 5x - 3) \div (x-1)$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -5 & -3 \\ & & 1 & 0 & -5 \\ \hline & 1 & 0 & -5 & \boxed{-8} \end{array}$$

$(x^3 - x^2 - 5x - 3) \div (x-1) =$ $x^2-5+\frac{-8}{x-1}$

Remainder Theorem:
If a polynomial function $P(x)$ is divided by a linear factor $x-a$ $\Rightarrow$ remainder = $P(a)$

1. If $P(x) = 3x^3 + 2x^2 - 1$, find $P(-2)$. Use substitution and syn. div. (find the remainder.

$P(-2) = 3(-2)^3 + 2(-2)^2 - 1$

$= -24 + 8 - 1$

$\boxed{P(-2) = -17}$

$$\begin{array}{r|rrrr} -2 & 3 & 2 & 0 & -1 \\ & & -6 & 8 & -16 \\ \hline & 3 & -4 & 8 & \boxed{-17} \end{array}$$

2. Find k so that $x-2$ is a factor of $x^4 - 2x^3 + kx + 6$. Use two different methods.

$P(2) = (2)^4 - 2(2^3) + k(2) + 6$

$0 = \cancel{16 - 16} + 2k + 6$

$0 = 2k + 6$

$-6 = 2k$

$\boxed{-3 = k}$

$$\begin{array}{r|rrrrr} 2 & 1 & -2 & 0 & k & 6 \\ & & 2 & 0 & 0 & 2k \\ \hline & 1 & 0 & 0 & k & \boxed{2k+6=0} \end{array}$$

$2k + 6 = 0$

$2k = -6$

$\boxed{k = -3}$

Summary: The remainder r, obtained in the synthetic division of $P(x)$ by $x-a$, provides the following information:

1. The remainder gives the value of P at $x =$ a. That is $r = P(a)$.
2. If $r =$ 0, then $(x -$ $a$$)$ is a factor of $P(x)$.
3. If $r =$ 0, then $(a, 0)$ is an x- intercept of the graph of P.

3. Find the remainder when $f(x) = x^6 - 10$ is divided by $g(x) = x - 2$ using two different methods.

I. Syn. Div.

$$\begin{array}{r|rrrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & 0 & -10 \\ & & 2 & 4 & 8 & 16 & 32 & 64 \\ \hline & 1 & 2 & 4 & 8 & 16 & 32 & \boxed{54} \end{array}$$

II. Substitution

$f(2) = 2^6 - 10 = \boxed{54}$

Which method was easier?

substitution!

Is $g(x)$ a factor of $f(x)$? Explain.

No! $r \neq 0$

Date ______________ 5.5 Notes Thms about Roots of Poly Eqns.

Rational Root Theorem:

If a polynomial function has integer coefficients, then its rational zeros can be found by using the rational roots theorem.

$p(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x^1 + a_0$

p = a factor of the constant (a_0). Then the possible rational roots $= \boxed{\dfrac{p}{q}}$ *

q = a factor of the leading coefficient (a_n).

1. Given $f(x) = 2x^3 - 3x^2 - 2x + 3$ (q: 2, p: 3)

   a) Identify all possible p and q values, and then all possible rational roots.

   $p: \pm 1, \pm 3$

   $q: \pm 1, \pm 2$

   $\boxed{\dfrac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}}$

   b) Use syn. div. to find one of the roots. Factor completely and find all zeros.

Try "1"!

$$\begin{array}{r|rrrr} 1 & 2 & -3 & -2 & 3 \\ & & 2 & -1 & -3 \\ \hline & 2 & -1 & -3 & \boxed{0} \end{array}$$

"1" works!

$f(x) = (x-1)(2x^2 - x - 3)$

$f(x) = (x-1)(2x-3)(x+1)$

these are in our list! $\rightarrow \boxed{x = 1, 3/2, -1}$

Helpful Hints: Find $f(1) = 2 - 3 - 2 + 3 = 0 \longrightarrow$ If $f(1) = 0 \Rightarrow$ $x - 1$ is a factor.

Find $f(-1) = 2(-1)^3 - 3(-1)^2 - 2(-1) + 3 = 0 \longrightarrow$ If $f(-1) = 0 \Rightarrow$ $x + 1$ is a factor.

2. Find the real zeros of $f(x) = 15x^3 - 32x^2 + 3x + 2$

   - Based on the Rational Root Theorem, *possible* zeros are =

     $p: \pm 1, \pm 2$

     $q: \pm 1, \pm 3, \pm 5, \pm 15$

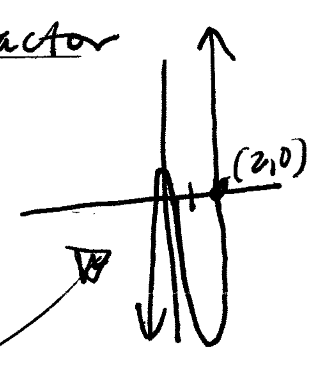
     $\boxed{\dfrac{p}{q} = \pm 1, \pm 2, \pm 1/3, \pm 2/3, \pm 1/5, \pm 2/5, \pm 1/15, \pm 2/15}$

   - Try the easiest, $x = 1$. $f(1) =$

     $15 - 32 + 3 + 2$

     $-17 + 3 + 2 = -12$ $\Rightarrow$ $x - 1$ is not a factor

   - Find the positive zero (integer) on your TI and use synthetic division to finish the problem.



$$\begin{array}{r|rrrr} 2 & 15 & -32 & 3 & 2 \\ & & 30 & -4 & -2 \\ \hline & 15 & -2 & -1 & 0 \end{array}$$

$f(x) = (x-2)(15x^2 - 2x - 1)$

$f(x) = (x-2)(5x+1)(3x-1)$

$\boxed{x = 2, -1/5, 1/3}$

Complex Conjugate Root Theorem

If $P(x)$ is a polynomial function with real coefficients and $a+bi$ is a root of $P(x)=0$

$\Rightarrow$ $a-bi$ is also a root.

Examples: Write the equation of a polynomial *function* in *factored form* and *standard form*, if …

Degree 3; zeros: 2 and $2i$, $-2i$

$f(x) = (x-2)(x-2i)(x+2i)$

$f(x) = (x-2)(x^2-4i^2)$

$\boxed{f(x) = (x-2)(x^2+4)}$

$\boxed{f(x) = x^3-2x^2+4x-8}$

Degree 4; zeros: i and $3i$, $-i$, $-3i$

$f(x) = (x-i)(x+i)(x-3i)(x+3i)$

$= (x^2-i^2)(x^2-9i^2)$

$\boxed{f(x) = (x^2+1)(x^2+9)}$

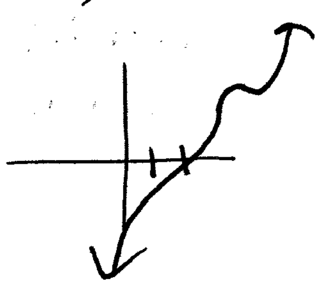
$\boxed{f(x) = x^4+10x^2+9}$

Explain what you know about the graphs of the related polynomial functions (*w/o using your TI)* for the above examples.

one real zero

$\Rightarrow$ one x-int.

ex:



no real zeros

$\Rightarrow$ no x-ints.

ex:

Date ______________ 5.6 Notes Fundamental Thrm of Algebra

The Fundamental Theorem of Algebra

If $P(x)$ is a polynomial function of degree $n \geq$ 1, then $P(x) = 0$ has exactly n complex roots.

[Remember the complex numbers include the real numbers.]

1. Find all the zeros of $8x^3 + 10x^2 - 11x + 2 = 0$.

a) List the *possible rational* roots

$p = \pm 1, \pm 2$

$q = \pm 1, \pm 2, \pm 4, \pm 8$

$\Rightarrow \boxed{\frac{p}{q} = \pm 1, \pm 1/2, \pm 1/4, \pm 1/8, \pm 2}$

b) Use your TI to find *one* of the roots. Then use synthetic division to factor the equation.

$$\begin{array}{r|rrrr} -2 & 8 & 10 & -11 & 2 \\ & & -16 & 12 & -2 \\ \hline & 8 & -6 & 1 & 0 \end{array}$$

c) Factor completely. Show your equation in factored form and find all the roots.

$(x+2)(8x^2-6x+1)=0$

$(x+2)(4x-1)(2x-1)=0$

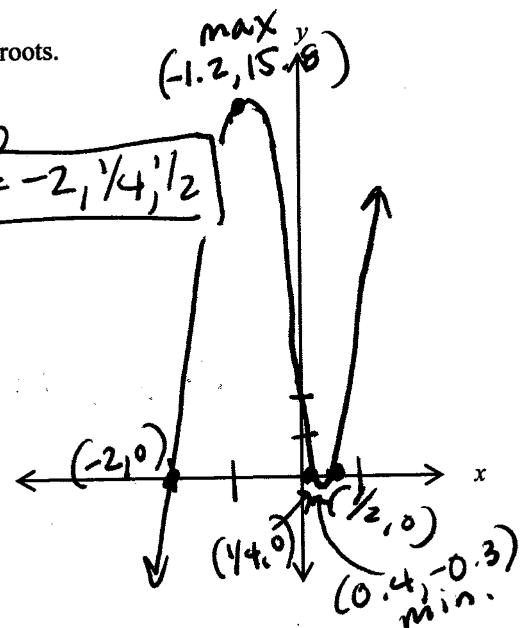
$\boxed{x = -2, 1/4, 1/2}$

d) Sketch the graph of the *related polynomial function* w/o your TI:

$P(x) = 8x^3 + 10x^2 - 11x + 2$

- At most 2 U turns (local max/min)
- Roots are $-2, 1/4, 1/2$
- Y-intercept is $(0,2)$
- End behavior as $x \to \infty, y \to \infty$ (rises rt)
  as $x \to -\infty, y \to -\infty$ (falls left)

Check with your TI.



e) Use your TI to find the *local extrema* values. Label on your graph.

---

2. a) Repeat for $-4x^3 + 2x^2 - x + 3 = 0$ $\boxed{\frac{p}{q} = \pm 1, \pm 3, \pm 1/2, \pm 1/4, \pm 3, \pm 3/2, \pm 3/4}$

$$\begin{array}{r|rrrr} 1 & -4 & 2 & -1 & 3 \\ & & -4 & -2 & -3 \\ \hline & -4 & -2 & -3 & 0 \end{array}$$

$(x-1)(-4x^2-2x-3)=0$

$x = \frac{2 \pm \sqrt{4-4(-4)(-3)}}{2(-4)}$

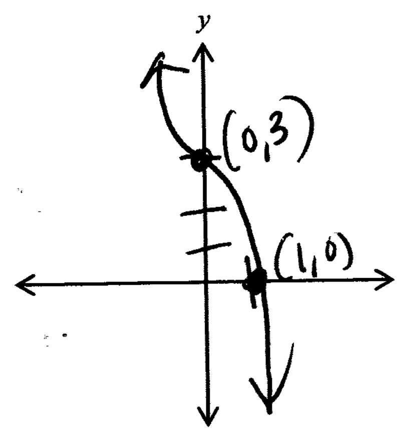
$= \frac{2 \pm \sqrt{4-48}}{-8}$

$= \frac{2 \pm \sqrt{-44}}{-8} = \frac{2 \pm 2i\sqrt{11}}{-8}$

$= \boxed{-\frac{1}{4} \pm \frac{i\sqrt{11}}{4}}$

b) Graph the *related polynomial function*, $P(x) = -4x^3 + 2x^2 - x + 3$

- Root(s) $1, -1/4 \pm i\sqrt{11}/4$
- Y-intercept: $(0,3)$
- End behavior: rises left, falls right
  $-4x^3$



Observation: The degree of the function is 3 and there are 3 roots to the equation above.

There is only 1 real root. The graph of the related polynomial function has only 1 x-intercept.

Date ______________ 5.7 Notes Binomial Theorem

Can you recognize the pattern in **Pascal's Triangle?**

1	Row 0
1 1	Row 1
1 2 1	Row 2
1 3 3 1	Row 3
1 4 6 4 1	Row 4
1 5 10 10 5 1	Row 5
1 6 15 20 15 6 1	Row 6

Binomial Theorem:
For every positive integer n,

$$(a+b)^n = P_0 a^n + P_1 a^{n-1} b + P_2 a^{n-2} b^2 + \dots + P_{n-1} a b^{n-1} + P_n b^n$$

Where $P_0, P_1, \dots, P_n$ are the numbers in the nth row of Pascal's Triangle.

1. What is the expansion of $(a+b)^6$? (row 6)

$$\boxed{a^6 + 6a^5 b + 15a^4 b^2 + 20a^3 b^3 + 15a^2 b^4 + 6ab^5 + b^6}$$

2. What is the expansion of $(3x-2)^5$? (row 5)

$$(3x)^5 + 5(3x)^4(-2) + 10(3x)^3(-2)^2 + 10(3x)^2(-2)^3 + 5(3x)(-2)^4 + (-2)^5$$

$$\boxed{243x^5 - 810x^4 + 1080x^3 - 720x^2 + 240x - 32}$$

3. What is the fourth term in $(2a+4b)^5$? row 5 $\longrightarrow$ 1 5 10 10

$$10(2a)^2(4b)^3 = \boxed{2560a^2b^3}$$

exponent add up to row #.

same # as term #.

** *Check out some of these websites to find interesting patterns and fun facts about Pascal's Triangle!*
- http://ptri1.tripod.com/
- http://www.mathsisfun.com/pascals-triangle.html
- http://mathforum.org/workshops/usi/pascal/pascal_numberpatterns.html

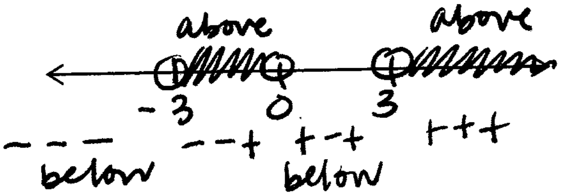
Date _______________ Supp. Notes Polynomial Inequalities

Solve. [Factor. Use cr. #'s and region testing!]

$x^3 - 9x > 0$

$x(x^2-9) > 0$ ← above!

$x(x-3)(x+3) > 0$



$\boxed{\{x: -3 < x < 0 \text{ or } x > 3\}}$

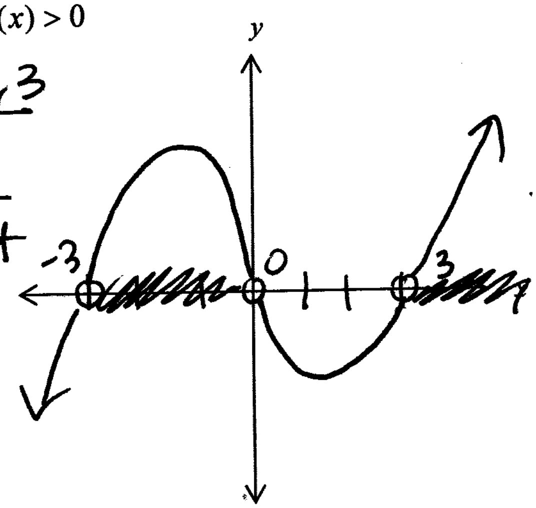
Graph the *related polynomial function* $P(x) = x^3 - 9x$.

Find the zeros and evaluate the end behavior. Sketch.

Use color to show $P(x) > 0$

$x = 0, -3, 3$

E.B: falls left rises right



Make the connection:

What information did the *solution to the inequality* give you regarding the graph of the *related polynomial function*?

The solution to the inequality refers to the part of the graph above the x-axis.

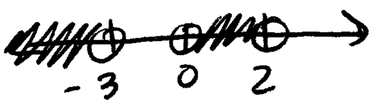
Solve. Factor, use cr. #'s and region testing. Check your answers by graphing the related func.on your TI. Sketch and use color.

a) $x^3 + x^2 < 6x$

$x^3 + x^2 - 6x < 0$

$x(x^2 + x - 6) < 0$

$x(x+3)(x-2) < 0$



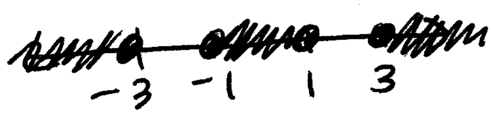
$\boxed{\{x: x < -3 \text{ or } 0 < x < 2\}}$

b) $x^4 + 9 \geq 10x^2$

$x^4 - 10x^2 + 9 \geq 0$

$(x^2-9)(x^2-1) \geq 0$

$(x-3)(x+3)(x-1)(x+1) \geq 0$



$\boxed{\{x: x \leq -3 \text{ or } -1 \leq x \leq 1 \text{ or } x \geq 3\}}$

c) $(x^2 - x - 6)(x^2 - 2x + 1) < 0$

$(x-3)(x+2)(x-1)^2 < 0$

$\boxed{\{x: -2 < x < 1 \text{ or } 1 < x < 3\}}$