Date ___________ Notes 6.1: Roots & Radical Exps.

It's the ...

Laws of Exponents

1. Product prop. $a^m \cdot a^n = a^{m+n}$
2. Quotient prop. $\frac{a^m}{a^n} = a^{m-n}$
3. Power of zero prop. $a^0 = 1$
4. Power of a power prop. $(a^m)^n = a^{mn}$
5. Power of a quotient prop. $\left(\frac{a}{b}\right)^m = a^m/b^m$
6. Power of a product prop. $(ab)^m = a^m b^m$

do First

*7. Power of a negative exponent prop. $a^{-m} = \frac{1}{a^m}$ $\frac{1}{a^{-m}} = a^m$

Examples:

1. $5^7 \cdot 5^3 = 5^{10}$
2. $\frac{5^7}{5^3} = 5^4$
3. $5^0 = 1$
4. $(5^7)^3 = 5^{21}$
5. $(5/7)^3 = 5^3/7^3$
6. $(5x)^3 = 5^3 x^3$
7. $5^{-3} = 1/5^3$

$\frac{1}{5^{-3}} = 5^3$

Negative Exponents

Find the reciprocal:

a. $3 \rightarrow 1/3$

b. $\frac{3}{4} \rightarrow 4/3$

c. $\left(\frac{3}{5}\right)^2 \rightarrow \frac{5^2}{3^2} = 25/9$

d. $2^3 \rightarrow \frac{1}{8}$

Use your TI to compute the following. Write answer as an improper fraction.

a. $3^{-1} = 1/3$

b. $\left(\frac{3}{4}\right)^{-1} = 4/3$

c. $\left(\frac{3}{5}\right)^{-2} = 25/9$

d. $2^{-3} = 1/8$

What does the negative exponent do? Reciprocal (fill in *7)

Problems – Simplify. Write answers with positive exponents only.

1. $3a^2\left(-4a^3b^5\right)^2\left(ab^7\right)^4$

$3a^2 \cdot 16a^6b^{10} \cdot a^4b^{28}$

$\boxed{48a^{12}b^{38}}$

2. $\left(\frac{4p^3q^3}{3pq^3}\right)^2 \cdot \frac{6p^4}{48q^7}$

$\left(\frac{4p^3q^3}{3}\right)^2 \cdot \frac{3p^4}{4q^7}$

$\frac{16p^6q^6}{9} \cdot \frac{3p^4}{4q^7}$

$\boxed{4p^{10}/}$

3. $\frac{\left(3x^2y^4\right)^3}{6\left(x^5y\right)^2}$

$\frac{27x^6y^{12}}{6x^{10}y^2}$

$\boxed{\frac{9y^{10}}{2x^4}}$

4. $-3x^2\left(4x^4+6x-5\right)$

$\boxed{-12x^6-18x^3+15x^2}$

Problems – Simplify and write with positive exponents only.

1. 3^{-4} $\frac{1}{3^4} = \boxed{\frac{1}{81}}$

2. $x^{-3} \cdot x$ $= \frac{x}{x^3} = \boxed{\frac{1}{x^2}}$

3. $\left(2u^5\right)^{-3}$ $= \frac{1}{(2u^5)^3} = \boxed{\frac{1}{8u^{15}}}$

4. $\frac{10a^{-3}b^2}{15a^3b^{-5}}$ $\boxed{\frac{2b^7}{3a^6}}$

5. $\left(\frac{3x^{-2}}{4x^{-5}y^7}\right)^{-2}$ $\left(\frac{4x^{-5}y^7}{3x^{-2}}\right)^2$ $\frac{16x^{-10}y^{14}}{9x^{-4}} = \frac{16y^{14}}{9x^6}$

6. $\frac{\left(2x^{-5}y^4\right)^{-3}}{\left(7xy^{-3}\right)^2}$ $= \frac{1}{(2x^{-5}y^4)^3(7xy^{-3})}$ $\frac{1}{8x^{-15}y^{12} \cdot 49x^2y^{-6}}$ $\boxed{\frac{x^{13}}{392y^6}}$

Key Concept: The n^{th} root

If $a^n = b$, with a and b real numbers and n a positive integer, then a is an n^{th} **root** of b.

If n is odd …

There is 1 real n^{th} root of b,

Denoted in radical form as $\sqrt[n]{b}$.

The only n^{th} root of 0 is 0.

If n is even …

- And b is positive, there are 2 real n^{th} roots of b. The positive root is the principal root and its symbol is $\sqrt[n]{b}$. The negative root is its opposite, or $-\sqrt[n]{b}$.
- And b is negative, there are no real roots of b.

Problems –Find each real root. (Note: we will NOT be using imaginary roots in this chapter)

1. $\sqrt{144}$ $= \boxed{12}$

2. $-\sqrt{25}$ $\boxed{-5}$

3. $\sqrt{-0.01}$ $\boxed{\varnothing}$

4. $\sqrt[3]{0.001}$ $= \sqrt[3]{1/1000}$ $= \frac{1}{10} = .1$

5. $\sqrt[4]{0.0081}$ $= \sqrt[4]{\frac{81}{10000}}$ $\frac{3}{10} = \boxed{.3}$

6. $\sqrt[3]{27}$ $= \boxed{3}$

7. $\sqrt[3]{-27}$ $\boxed{-3}$

8. $\sqrt{0.09}$ $\boxed{.3}$

Find the two real solutions of each equation.

1. $\sqrt{x^2} = \sqrt{4}$ $\boxed{x = \pm 2}$

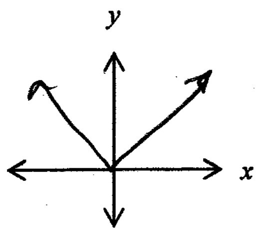
2. $\sqrt[4]{x^4} = \sqrt[4]{81}$ $\boxed{x = \pm 3}$

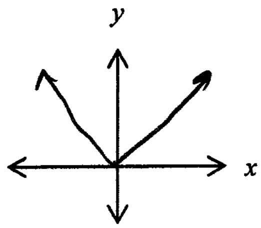
3. $\sqrt{x^2} = \sqrt{0.16}$ $\boxed{x = \pm .4}$

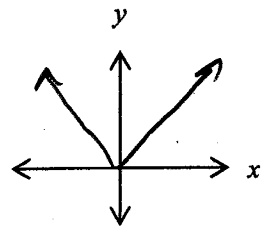
4. $\sqrt{x^2} = \sqrt{\frac{16}{49}}$ $\boxed{x = \pm \frac{4}{7}}$

Properties of Radicals.

$y=\sqrt{x^2}$ = $y=\sqrt[4]{x^4}$ =   What do you think $y=\sqrt[n]{x^n}$ will look like if n is an even integer? $y=|x|$







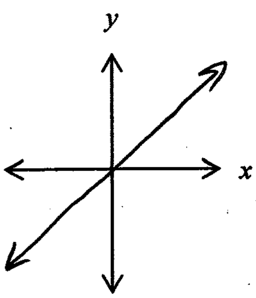
When simplifying even root radicals, the answer must be a positive number, so you need absolute value.

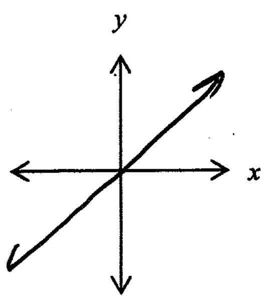
Properties of Radicals: For any real number a,

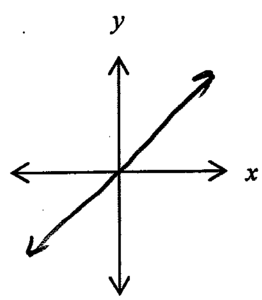
$\sqrt[n]{a^n}=|a|$ if n is a positive even integer

Use absolute value when you have even even odd. Ex: $\sqrt[4]{x^{12}}=|x^3|$

$y=\sqrt[3]{x^3}=x$ $y=\sqrt[5]{x^5}=x$ What do you think $y=\sqrt[n]{x^n}$ will look like if n is an odd integer? $y=x$







When simplifying odd root radicals, the answer can be a positive or negative number, so you do not need absolute value.

Properties of Radicals: For any real number a,

$\sqrt[n]{a^n}=a$ if n is a positive odd integer

Simplify the following. Assume the variables represent *real numbers.*

1. $\sqrt{50x^6z^4}$

$\sqrt{25\cdot 2\cdot x^6z^4}$

$5|x^3|z^2\sqrt{2}$

2. $\sqrt[3]{24x^5y^6}$

$\sqrt[3]{8\cdot 3x^5y^6}$

$2xy^2\sqrt[3]{3x^2}$

3. $\sqrt[4]{80x^5t^8y^{14}}$

$\sqrt[4]{16\cdot 5\cdot x^4\cdot x\cdot t^8\cdot y^{12}\cdot y^2}$

$\boxed{2xt^2|y^3|\sqrt[4]{5y^2}}$

4. $\sqrt[5]{-96x^5y^{10}}$

$\sqrt[5]{-32\cdot 3x^5y^{10}}$

$\boxed{-2xy^2\sqrt[5]{3}}$

Date ______________ Notes 6.2 Multi & Div. Radical Exps.

Review. Simplify :

1. $\sqrt[6]{x^{24}z^{18}}$ $\boxed{x^4|z^3|}$

2. $\sqrt{x^{12}y^7z^{10}}$ $\boxed{x^6y^2|z^5|\sqrt{y}}$

3. $\sqrt[3]{x^3y^{12}}$ $\boxed{xy^4}$

4. $\sqrt[5]{x^{10}y^{18}}$ $\boxed{x^2y^3\sqrt[5]{y^3}}$

Property – Products of Radicals:

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\sqrt[n]{a}\cdot\sqrt[n]{b}$ = $\sqrt[n]{ab}$

Simplify the following. Assume the variables represent *positive numbers only*.

5. $\sqrt[5]{16x^8}\cdot\sqrt[5]{2x^4}$

$\sqrt[5]{32x^{12}}$

$\boxed{2x^2\sqrt[5]{x^2}}$

6. $\sqrt[4]{8x^2y^5}\cdot\sqrt[4]{8x^2y}$

$\sqrt[4]{64x^4y^6}$

$\sqrt[4]{16\cdot4x^4y^6}$

$\boxed{2xy\sqrt[4]{4y^2}}$

7. $\sqrt{45x^5y^3}\cdot\sqrt{35xy^4}$

$\sqrt{9\cdot5\cdot5\cdot7x^6y^7}$

$3\cdot5x^3y^3\sqrt{7y}$

$\boxed{15x^3y^3\sqrt{7y}}$

Property – Quotients of Radicals:

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then $\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ = $\sqrt[n]{\frac{a}{b}}$

Simplify the following. Assume the variables represent *positive numbers only*. If necessary, rationalize all denominators (no radical in a denominator; no denominator in a radical).

1. $\frac{\sqrt{50x^6}}{\sqrt{2x^4}}$

$\sqrt{25x^2}$

$\boxed{5x}$

2. $\frac{\sqrt[3]{96x^2y^5z^4}}{\sqrt[3]{4xy}} = \sqrt[3]{24xy^4z^4}$

$\sqrt[3]{8\cdot3xy^4z^4} = \boxed{2yz\sqrt[3]{3xyz}}$

To rationalize a denominator with cube roots:

1. Factor the denominator.
2. Determine the missing factors to make a perfect cube.
3. Multiply the numerator and denominator by the cube root of these factors.

3. $\frac{\sqrt[3]{7x}}{\sqrt[3]{5y^2}}\cdot\frac{\sqrt[3]{5^2y}}{\sqrt[3]{5^2y}}$

$\frac{\sqrt[3]{7\cdot25xy}}{\sqrt[3]{5^3y^3}} = \boxed{\frac{\sqrt[3]{175xy}}{5y}}$

4. $\sqrt[3]{\frac{5x^2z}{12xy^2z^2}}\cdot\frac{\sqrt[3]{5x}}{\sqrt[3]{2^2\cdot3\cdot y^2z}}$

$\frac{\sqrt[3]{5x}}{\sqrt[3]{2^2\cdot3xy^2z^2}}\cdot\frac{\sqrt[3]{2\cdot3^2\cdot yz^2}}{\sqrt[3]{2\cdot3^2\cdot y\cdot z^2}}$

$\sqrt[3]{90xyz^2}$

Date ______________ Notes 6.3 Binomial, Radical Exps

Review:

Simplify:

a) $\sqrt{125} = \sqrt{25 \cdot 5}$

$\boxed{5\sqrt{5}}$

b) $2\sqrt{27}$

$2 \cdot 3\sqrt{3}$

$\boxed{6\sqrt{3}}$

c) $5\sqrt{18} - 4\sqrt{50} + 2\sqrt{75}$

$5 \cdot 3\sqrt{2} - 4 \cdot 5\sqrt{2} + 2 \cdot 5\sqrt{3}$

$15\sqrt{2} - 20\sqrt{2} + 10\sqrt{3}$

$\boxed{-5\sqrt{2} + 10\sqrt{3}}$

---

d) $(5 + \sqrt{125}) + (-10 + 10\sqrt{5})$

$5 + 5\sqrt{5} - 10 + 10\sqrt{5}$

$\boxed{-5 + 15\sqrt{5}}$

e) $(4 + \sqrt{7}) - (-3 + 2\sqrt{7})$

$4 + \sqrt{7} + 3 - 2\sqrt{7}$

$\boxed{7 - \sqrt{7}}$

f) $3\sqrt{5}(-\sqrt{20} + 2)$

$-3\sqrt{5 \cdot 5 \cdot 4} + 6\sqrt{5}$

$\boxed{-30 + 6\sqrt{5}}$

---

g) $(-6 + \sqrt{5})(-4 + 2\sqrt{5})$

$24 - 12\sqrt{5} - 4\sqrt{5} + 10$

$\boxed{34 - 16\sqrt{5}}$

h) $\frac{5}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}}$

$\frac{5\sqrt{15}}{15} = \boxed{\frac{\sqrt{15}}{3}}$

i) $(6 - \sqrt{12})(6 + \sqrt{12})$

$36 - 12 = \boxed{24}$

Use the **conjugate** to rationalize each denominator:

j) $\frac{12}{\sqrt{5} - 1} \cdot \frac{\sqrt{5} + 1}{\sqrt{5} + 1}$

$\frac{12\sqrt{5} + 12}{5 - 1} = \frac{12\sqrt{5} + 12}{4}$

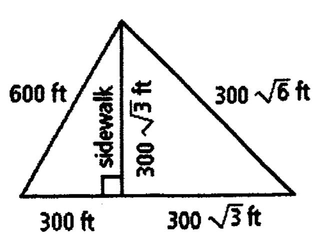
$\boxed{3\sqrt{5} + 3}$

k) $\frac{2\sqrt{7}}{\sqrt{3} - \sqrt{5}} \cdot \frac{\sqrt{3} + \sqrt{5}}{\sqrt{3} + \sqrt{5}} = \frac{2\sqrt{21} + 2\sqrt{35}}{3 - 5}$

$= \frac{2\sqrt{21} + 2\sqrt{35}}{-2}$

$= \boxed{-\sqrt{21} - \sqrt{35}}$

Application: A park in the shape of a triangle has a sidewalk dividing it into two parts.



a. If a man walks around the perimeter of the park, how far will he walk?

$600 + 300\sqrt{6} + 300\sqrt{3} + 300$

$\boxed{900 + 300\sqrt{6} + 300\sqrt{3} \text{ ft}}$

b. What is the area of the park?

$\frac{1}{2}(300 + 300\sqrt{3})(300\sqrt{3})$

$\frac{1}{2}(90000\sqrt{3} + 90000 \cdot 3) = \frac{1}{2}(900000\sqrt{3} + 270000)$

Date ____________ Notes 6.4 Rational Exponents

Rational Exponents

Use your calculator to compute the following. [Note: you need parentheses around the exponents.]

a. $9^{\frac{1}{2}} =$ 3 b. $16^{\frac{1}{2}} =$ 4 c. $49^{\frac{1}{2}} =$ 7 What does the one-half exponent do?

Square root

d. $8^{\frac{1}{3}} =$ 2 e. $64^{\frac{1}{3}} =$ 4 f. $125^{\frac{1}{3}} =$ 5 What does the one-third exponent do?

Cube root

What do you think will happen with $125^{\frac{2}{3}}$?

Cube root, squared

Key Concept: Rational Exponents

If the n^{th} root of a is a real number, m is an integer, and $\frac{m}{n}$ is in lowest terms then

$a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{(a^m)}$

Problems – Evaluate without your calculator. You may use your calculator to check your answers.

1. $\sqrt[4]{625} = \sqrt[4]{25 \cdot 25}$ $\boxed{5}$

2. $16^{\frac{3}{2}} = \left(\sqrt[2]{16}\right)^3 = 4^3 = \boxed{64}$

3. $25^{-\frac{1}{2}}$ $\frac{1}{\sqrt{25}}$ $\boxed{1/5}$

4. $\left(\frac{4}{9}\right)^{-\frac{3}{2}} = \left(\frac{9}{4}\right)^{3/2} = \frac{\left(\sqrt[2]{9}\right)^3}{\left(\sqrt[2]{4}\right)^3}$ $3^3/2^3 = \boxed{\frac{27}{8}}$

Simplify each expression.

5. $(-5)^{\frac{1}{3}} \cdot (-5)^{\frac{1}{3}} \cdot (-5)^{\frac{1}{3}}$

$(-5)^1 = \boxed{-5}$

6. $7^{\frac{1}{2}} \cdot 28^{\frac{1}{2}}$

$\sqrt{7 \cdot 7 \cdot 4} = \boxed{14}$

7. $12^{\frac{1}{3}} \cdot 45^{\frac{1}{3}} \cdot 50^{\frac{1}{3}}$

$\sqrt[3]{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 2}$

$2 \cdot 3 \cdot 5 = \boxed{30}$

Write each expression in radical form.

8. $x^{\frac{4}{3}}$

$\left(\sqrt[3]{x}\right)^4$

9. $t^{-\frac{2}{7}}$

$\frac{1}{\left(\sqrt[7]{t}\right)^2}$

10. $a^{-1.6} = a^{-16/10}$

$= a^{-8/5}$

$= \frac{1}{\left(\sqrt[5]{a}\right)^8}$

Write each expression in exponential form.

11. $\sqrt{x^3}$

$x^{3/2}$

12. $\sqrt[3]{m}$

$m^{1/3}$

13. $\sqrt[3]{2y^2}$

$(2y^2)^{1/3}$

Simplify each expression.

$\left(\frac{27x^6}{64y^4}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{27x^6}{64y^4}}$

$\frac{3x^2}{4y\sqrt[3]{y}} \cdot \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} = \boxed{\frac{3x^2\sqrt[3]{y^2}}{4y^2}}$

$\left(\frac{x^{-\frac{1}{3}}y}{x^{\frac{2}{3}}y^{-\frac{1}{2}}}\right)^2 = \left(\frac{y^{3/2}}{x}\right)^2$

$\boxed{\frac{y^3}{x^2}}$

$\left(\frac{12x^8}{75y^{10}}\right)^{\frac{1}{2}}$ $\frac{\sqrt[2]{2 \cdot 2 \cdot \cancel{3}\, x^8}}{\sqrt[2]{5 \cdot 5 \cdot \cancel{3}\, y^{10}}}$

$\boxed{\frac{2x^4}{5y^5}}$

Date ____________ Notes 6.5 Solving Sq. Root & other Radicals.

Definitions:

Example: $\sqrt[3]{2x+5}$

- Radical symbol: $\sqrt{}$
- Radicand: $2x+5$
- Index: 3

Reciprocals: Two numbers are reciprocals if their product is 1

Examples: 3 and $1/3$

$-\frac{7}{4}$ and $-4/7$

Steps for Solving Radical Equations:

1. Isolate the radical
2. Raise both sides of the equation by the index
3. Solve for the variable
4. Check your answers to see if it's correct
   False answers are call extraneous solutions

Solve a radical equation *algebraically*. Be sure to check your answers for extraneous solutions !!!

a) $\left(\sqrt{x-3}\right)^2 = (1)^2$

$x-3=1$

$x=4$

b) $\left(4\sqrt{x}\right)^2 = \left(\sqrt{x-5}\right)^2$

$16x = x-5$

$15x = -5$

$x = -1/3$

$\varnothing$

c) $\left(\sqrt[3]{x+4}\right)^3 = \left(\sqrt[3]{3x-6}\right)^3$

$x+4 = 3x-6$

$10 = 2x$

$\boxed{x=5}$

d) $x = 5+\sqrt{3x-5}$

$(x-5)^2 = \left(\sqrt{3x-5}\right)^2$

$x^2-10x+25 = 3x-5$

$x^2-13x+30=0$

$(x-10)(x-3)=0$

$10, \cancel{3}$

$x=10$

e) $\sqrt{2x+5} - 2\sqrt{2x} = 1$

$\left(\sqrt{2x+5}\right)^2 = \left(1+2\sqrt{2x}\right)^2$

$(2x+5) = (1+2\sqrt{2x})(1+2\sqrt{2x})$

$2x+5 = 1+2\sqrt{2x}+2\sqrt{2x}+8x$

$(-6x+4)^2 = \left(4\sqrt{2x}\right)^2$

$36x^2-48x+16 = 32x$

$36x^2-80x+16=0$

$9x^2-20x+4=0$

$(9x-2)(x-2)=0$

$x = 2/9, \cancel{2}$

$\boxed{2/9}$

Solve a radical equation by raising each side to the reciprocal power:

Example: $2(x+3)^{\frac{2}{3}}=8$

$(x+3)^{2/3}=4$

$\left((x+3)^{2/3}\right)^{3/2}=(4)^{3/2}$

$x+3=\pm(\sqrt{4})^3$

$x+3=\pm 8$

$x=-11, 5$

1. Isolate the radical term.

2. Raise each side to the reciprocal power.

Since the numerator of $\frac{2}{3}$ is even, $\left((x+3)^{\frac{2}{3}}\right)^{\frac{3}{2}}=|x+3| \therefore$ need $\pm$!

3. Solve for x.

Solve:

1. $4x^{\frac{1}{2}}-5=27$

$4x^{1/2}=32$

$x^{1/2}=8$

$(x^{1/2})^2=8^2$

$\boxed{x=64}$

2. $(x-2)^{\frac{2}{3}}-4=5$

$\left[(x-2)^{2/3}\right]^{3/2}=(9)^{3/2}$

$x-2=\pm(\sqrt{9})^3$

$x-2=\pm 27$

$\boxed{x=29, -25}$

3. $4x^{\frac{3}{2}}-5=103$

$4x^{3/2}=108$

$x^{3/2}=27$

$(x^{3/2})^{2/3}=27^{2/3}$

$x=(\sqrt[3]{27})^2$

$x=3^2=\boxed{9}$

4. The formula $A=6V^{\frac{2}{3}}$ relates the surface area A, in square units, of a cube to the volume V, in cubic units. What is the volume of a cube with surface area 486 in.^{2}?

$486=6V^{2/3}$

$81=V^{2/3}$

$81^{3/2}=(V^{2/3})^{3/2}$

$V=(\sqrt{81})^3=9^3=\boxed{729 \text{ in}^3}$

Date: ___________ Notes 6.6 Function Operations

Review:

Relation: set of ordered pairs

Domain: set of all inputs or x-values

Range: set of all outputs or y-values

Ex: { (-3, 5), (0, -7), (4, 5), (6, -9)}

Ex: D = {-3, 0, 4, 6}

Ex: R = {-9, 7, 5}

Function:
A ***special*** relation such that each input or X value corresponds to exactly 1 output or y value.

Function notation: $f(x)$ "f of x" → x is the input
$[f(x)=y]$ f(x) is the output

Let $f(x)=5-3x$, $g(x)=-x^2+6x$ and $k(x)=\dfrac{x}{2x+5}$. Find

a) $k(-3)$ $\dfrac{-3}{2(-3)+5} = \dfrac{-3}{-6+5} = \boxed{3}$

b) $f(3t) = 5-3(3t)$
$\boxed{5-9t}$

c) $g(5x)$
$-(5x)^2+6(5x)$
$\boxed{-25x^2+30x}$

d) $f(a)-f(b)$
$5-3a-(5-3b)$
$\boxed{-3a+3b}$

e) $f(x+h)$
$5-3(x+h)$
$\boxed{5-3x-3h}$

f) $g(x+3)$
$-(x+3)^2+6(x+3)$
$-(x^2+6x+9)+6x+18$
$-x^2-6x-9+6x+18$
$\boxed{-x^2+9}$

g) $k(x+h)$
$\dfrac{x+h}{2(x+h)+5}$
$\dfrac{x+h}{2x+2h+5}$

Domain Restrictions:

Radicals – What can't you have under the radical? a neg # → inside ≥ 0

Fractions – What can't you have in a fraction? denom ≠ 0

Problems – Find the domain for the functions algebraically.

1. $f(x)=\sqrt{x-3}$
$x-3\ge 0$
$x\ge 3$
$D: \{x: x\ge 3\}$

2. $g(x)=\dfrac{1}{x-3}$
$x-3\ne 0$
$x\ne 3$
$D=\{x \in R\}$

3. $h(x)=\sqrt{2x+5}$
$2x+5\ge 0$
$2x\ge -5$
$x\ge -5/2$
$D=\{x: x\ge -5/2\}$

4. $k(x)=\dfrac{x}{2x+5}$
$2x+5\ne 0$
$D=\{x: x\in R$ & $x\ne -5/2\}$

5. $j(x)=3x+5$
$D=\{x: x\in R$

Operations with functions – Find each new function, and simplify your answers. State any domain restrictions.

Let $f(x)=4x^2+6x-9$, $g(x)=5x-2$, and $h(x)=x+3$.

a) $f+g=(f+g)(x)$

$4x^2+6x-9+5x-2$

$\boxed{4x^2+11x-11}$

b) $h-f$ $\quad x+3-(4x^2+6x-9)$

$\boxed{-4x^2-5x+12}$

c) $g \cdot h$

$(5x-2)(x+3)$

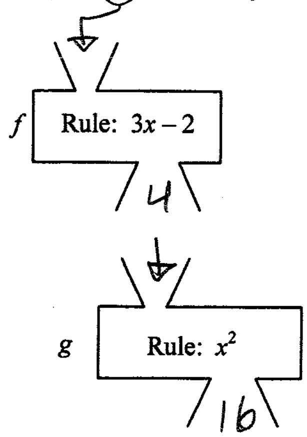
$\boxed{5x^2+13x-6}$

d) $\frac{g}{h} = \frac{5x-2}{x+3}$

$D: \{x : x \in R, x \neq -3\}$

Composition of Functions

Let $f(x)=3x-2$ and $g(x)=x^2$.
Using function machines, start with an input of 2 in function f.



What did we do?
$f(2)=4$ $\quad g(f(2))=16$
$g(4)=16$

Try… $g(f(-3))$

$f(-3)=3(-3)-2$
$=-11$
$g(-11)=(-11)^2=121$

Composition of Functions is when the output of one function becomes the input of the next function.

Notation: $f \circ g = (f \circ g)(x) = f(g(x))$

Let $f(x)=2x+3$, $g(x)=4x^2$, $h(x)=5x-2$. Find

1. $(g \circ h)(3)$

$g(h(3))$
$5(3)-2=13$
$g(13)=4(13)^2$
$=4(169)$
$=\boxed{676}$

2. $h \circ f = h(f(x))$

$h(2x+3)$
$5(2x+3)-2$
$10x+15-2$
$\boxed{10x+13}$

3. $(g \circ f)(x)$

$g(2x+3)$
$=4(2x+3)^2$
$=4(4x^2+12x+9)$
$\boxed{16x^2+48x+36}$

Application of Composite Functions

A computer store is offering a \$40 rebate along with a 20% discount. Let x = original price of an item.

Write a function that represents the sale price after the 20% discount.

$D(x) = 0.8x$

Find $(D \circ R)(x) = D(x-40)$

$= .8(x-40)$

$= \boxed{.8x - 32}$

Write a function that represents the sale price after the \$40 rebate

$R(x) = x - 40$

$(R \circ D)(x) = R(.8x)$

$= \boxed{.8x - 40}$

Explain in complete sentences what each one represents. As the customer, which one gives you the better deal?

with $D(R(x))$ the customer gets \$40 rebate, then 20%

with $R(D(x))$ the customer gets 20% off, then a \$40 rebate

★ $R(D(x))$ is the better deal

Date ____________________ Notes 6.7 Inverse Relations / Functions

Inverses of Functions

Explain what the net result is when a function is followed by its inverse. Write a physical example.

They undo each other → square / square root

add / subtract

Suppose function f converts # of feet to the # of inches.

$f(x) = 12x$

Give some ordered pairs that belong to $f(x)$:

$(1, 12), (2, 24), (3, 36)$

Suppose function g converts # of inches to the # of feet.

$g(x) = \frac{1}{12}x$

Give some ordered pairs that belong to $g(x)$:

$(12, 1), (24, 2), (36, 3)$

Are f and g inverses? yes What do you notice about the ordered pairs? x & y are switched

> The **domain** of the function = range of its inverse.
>
> The **range** of the function = domain of its inverse.

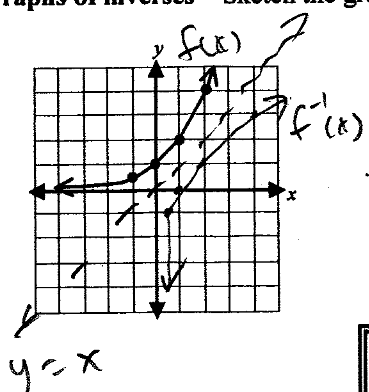
Note: $R^{-1} \neq \frac{1}{R}$

Given the relation $R = \{(1,2), (4,-2), (3,2)\}$. Is R a function? yes

Write the ordered pairs for the inverse of R: $R^{-1} = \{(2, 1), (-2, 4), (3, 2)\}$ Is R^{-1} a function? no

---

Graphs of inverses – Sketch the graph of the inverse. Hint: Switch the x and y values of the ordered pairs.



Key points on f: $(-1, \frac{1}{2}), (0, 1), (1, 2), (2, 4)$

Key points on f^{-1}: $(\frac{1}{2}, -1), (1, 0), (2, 1), (4, 2)$

f: D = $\{-1, 0, 1, 2\}$

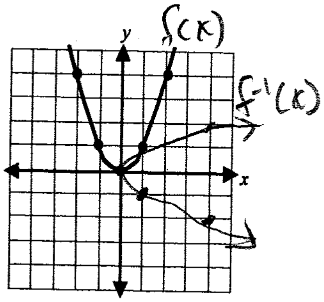
R = $\{\frac{1}{2}, 1, 2, 4\}$

f^{-1}: D = $\{\frac{1}{2}, 1, 2, 4\}$

R = $\{-1, 0, 1, 2\}$

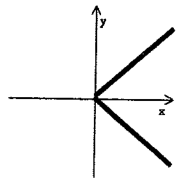
Draw in the line $y = x$ using a colored pencil. What do you notice?

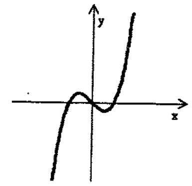
> Functions and their inverses are symmetric over the line $y = x$.

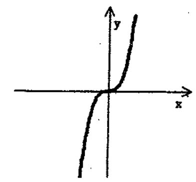


Sketch the inverse. Is the inverse a function? NO

The Vertical Line test determines whether a relation is a function. What kind of line will determine if the ***inverse of a relation*** is a function? horizontal line







Use the VLT to determine if the relation is a function and the HLT to determine if the inverse of the relation is a function.

Relation	no	yes	yes
Inverse	yes	NO	yes

The equation for the inverse, algebraically

1. $f(x) = 2x - 4$, find $f^{-1}(x)$

$y = 2x - 4$

$x = 2y - 4$

$x + 4 = 2y$

$\frac{x+4}{2} = y$

$f^{-1}(x) = \frac{1}{2}x + 2$

Note: $f^{-1}(x) \neq \frac{1}{f(x)}$

2. $g(x) = -\frac{1}{5}x + 2$, find $g^{-1}(x)$

$y = -\frac{1}{5}x + 2$

$x = -\frac{1}{5}y + 2$

$x - 2 = -\frac{1}{5}y$

$-5x + 10 = y$

$g^{-1}(x) = -5x + 10$

Finding the inverse *algebraically*:
1. Replace f(x) with y
2. Switch x to y and y to x
3. Solve for y
4. Replace y with $f^{-1}(x)$

Composition and Inverses

Let $f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x + 2$. What do we already know about f and g? They are inverses

Find $(f \circ g)(x) = f(\frac{1}{2}x + 2)$

$= 2(\frac{1}{2}x + 2) - 4$

$= x + 4 - 4$

$= \boxed{x}$

$(g \circ f)(x) = g(2x - 4)$

$= \frac{1}{2}(2x - 4) + 2$

$= x - 2 + 2$

$= \boxed{x}$

The following **property of inverses** is used to determine if two functions are inverses of each other.

$(f \circ g)(x) = (g \circ f)(x) =$ x $\Leftrightarrow$ f and g are inverses

Problem – Use the **property of inverses** to determine whether f and g are inverses: $f(x) = \frac{x-3}{5}$ and $g(x) = 5x + 3$.

$(f \circ g)(x) = f(5x + 3)$

$= \frac{5x + 3 - 3}{5}$

$= \frac{5x}{5} = \boxed{x}$

$(g \circ f)(x) = g(\frac{x-3}{5})$

$= 5(\frac{x-3}{5}) + 3$

$= x - 3 + 3$

$= \boxed{x}$

Since $(f \circ g)(x) =$ x and $(g \circ f)(x) =$ x

$\Rightarrow$ f and g are inverses.