Date ______________ Notes 7.1/7.2: Exploring Exponential Models & properties

1.1A

A population of 12 wabbits doubles every year. Find the population of the wabbits in years 0 through 4. Find the population in the nth year.

Year	Population	
0	12	$=12$
1	24	$=12(2)^1$
2	48	$=12(2)^2$
3	96	$=12(2)^3$
4	192	$=12(2)^4$
n	—	$=12(2)^n$

The expression $12(2)^n$ is called an exponential expression, because the exponent, n, is a variable and the base 2 is a fixed number. The base is called a multiplier.

$y = a \cdot b^x$, where $a =$ initial amount and $b =$ multiplier

To determine the **multiplier** when given a %:

- **exponential growth:** add the growth rate to 100%
- **exponential decay:** subtract the decay rate from 100%

Find the **multiplier** for the following.

a) 7% growth

100% +7% = 107% or 1.07

b) 6% decay

100% −6% = 94% or .94

c) 1.5% growth

100% +1.5% = 101.5% or 1.015

d) 8.2% decay

100% −8.2% = 91.8% or .918

Use your calc. to compute (3 dec. pl.) if $x = \frac{2}{3}$ and $y = 3.4$:

a) $42(2)^{x-1} \approx$ 33.335

b) $0.8^{3y} \approx$ 0.103

Applications: $A(t) = a(1+r)^t$

1. The initial population of bacteria in a lab test is 400. The number of bacteria *doubles* every hour.
   a) Multiplier = 2
   b) Predict the bacteria population at the end of 3 hours.
   $400(2)^3 = 3200$ bacteria
   c) Predict the bacteria population at the end of 5 hours.
   $400(2)^5 = 12{,}800$ bacteria

2. The initial population of bacteria in a lab test is 400. The number of bacteria *doubles* every 30 minutes.
   a) Multiplier = 2
   b) Predict the bacteria population at the end of 3 hours.
   $400(2)^6 = 25{,}600$ bacteria
   c) Predict the bacteria population at the end of 5 hours.
   $400(2)^{10} = 409{,}600$ bacteria

3. A certain medicine is *eliminated* from the bloodstream at a rate of 18% every minute. [Hint: exp. growth or decay?]
   a) Multiplier = 100% − 18% = 82% or 0.82
   b) If 60 mg of the medication is in the bloodstream, predict the amount of medicine remaining after 20 minutes. $60(.82)^{20} \approx$ 1.134 mg

4. Suppose you invest $200 into an account that earns 6.5% interest, compounded annually. [growth or decay?]
   a) Multiplier = 100% + 6.5% = 106.5% or 1.065
   b) Write an expression that will predict the balance in the account after t years. $200(1.065)^t$
   c) Find the balance after 7 years. $200(1.065)^7 \approx$ $310.80
   d) *When* is the balance at least $1000? Set up an equation: $200(1.065)^t = 1000$
   $1.065^t = 5$
   — ??? —
   Solve graphically. $y_1 = 1.065^x$
   $y_2 = 5$
   The balance will be at least $1000 after 26 years.

← I going to show them this with the calculator!
window [0, 30] by [0, 7]

7.1B compound interest

Suppose you invest \$100 in an account that earns 5% interest *compounded annually* (once at the end of the year.) How much is your investment worth after 12 years? *Growth* or *decay*?

Multiplier = 100% + 5% or 1 + .05

$A = 100(1+.05)^{12} \approx \$179.59$

Suppose you invest \$100 at 5% annual interest, compounded semi-annually. You do not get 5% interest every 6 months.

You get $\frac{.05}{2}$ every 6 months.

# of years: t	Amount of \$: A
0	\$100
6 months = 0.5 years	$100(1+\frac{.05}{2})^1$
1 year	$100(1+\frac{.05}{2})^2$
1.5 years	$100(1+\frac{.05}{2})^3$
2 years	$100(1+\frac{.05}{2})^4$
...	...
t years	$100(1+\frac{.05}{2})^{2t}$

Compound Interest Formula: The total amount of an investment, A, is

$A = P(1+\frac{r}{n})^{nt}$

where...

P = principal (initial amount)

r = interest rate (decimal)

n = # of times interest is compounded per year

t = # of years

Find the amount of money you will have if you invest \$500 for 8 years @ 7% annual interest, compounded....

a) annually $n = 1$

$A = 500(1+\frac{.07}{1})^{(1)(8)} \approx \$859.09$

b) quarterly $n = 4$

$A = 500(1+\frac{.07}{4})^{(4\cdot 8)} \approx \$871.11$

c) monthly $n = 12$

$A = 500(1+\frac{.07}{12})^{(12\cdot 8)} \approx \$873.91$

d) daily $n = 365$

$A = 500(1+\frac{.07}{365})^{(365\cdot 8)} \approx \$875.29$

e) *continuously...* $A = Pe^{rt}$

$A = 500e^{(.07)(8)} \approx \$875.34$

Compounded...

Annually:	$n = 1$
Semiannually:	$n = 2$
Quarterly:	$n = 4$
Monthly:	$n = 12$
Weekly:	$n = 52$
Daily:	$n = 365$

Compound Continuously

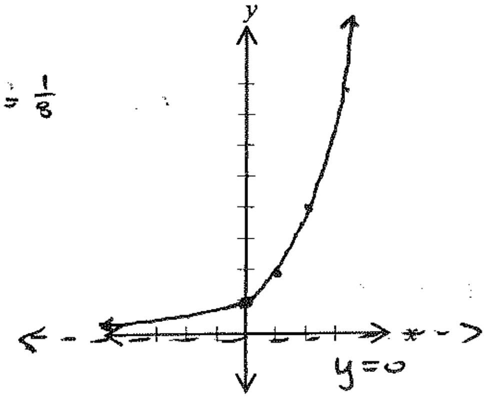
$A = Pe^{rt}$

Exponential Function: $f(x) = b^x$

where $b > 0$ and $b \neq 1$. $b =$ base

Graph $f(x) = 2^x$.

x	y
-3	$2^{-3} = \frac{1}{8}$
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4
3	8

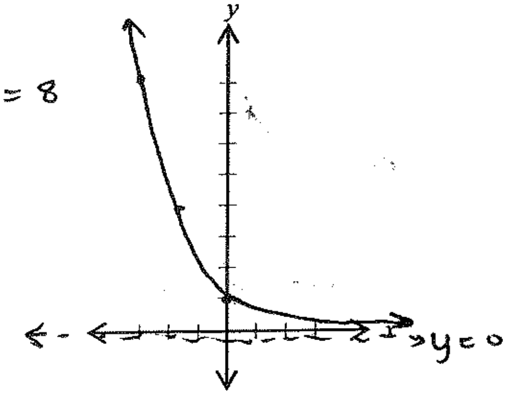


Conclusions: exp. growth ($b > 1$)

1. Domain = { $x \in \mathbb{R}$ } Range = { $y > 0$ }
2. y-int: (0 , 1)
3. graph is above the x-axis
4. graph is increasing
5. Horiz. asymptote at $y=0$

Graph $f(x) = \left(\frac{1}{2}\right)^x$

x	y
-3	$(1/2)^{-3} = 8$
-2	4
-1	2
0	1
1	$1/2$
2	$1/4$
3	$1/8$



Conclusions: exp decay ($0 < b < 1$)

1. Domain = { $x \in \mathbb{R}$ } Range = { $y > 0$ }
2. y-int: (0 , 1)
3. graph is above the x-axis
4. graph is decreasing
5. Horiz. asymptote at $y=0$

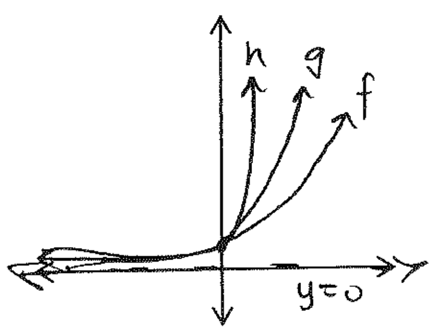
Use your calculator to graph:

$f(x) = 1.2^x$,

$g(x) = 2^x$,

$h(x) = 3^x$

Sketch and label each graph.
(use different colors to help)

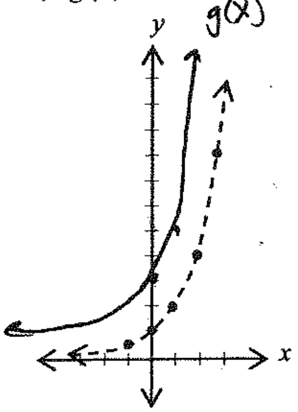


Conclusion:

Larger base $\Rightarrow$ faster growth

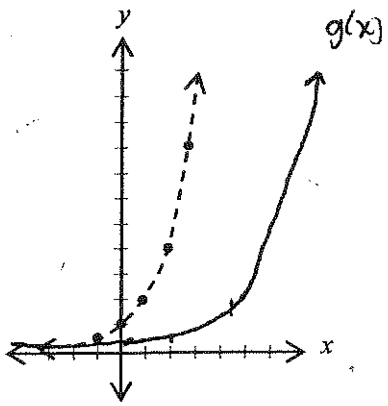
Describe the transformations from $f(x) = 2^x$ and graph.

a) $g(x) = 3 \cdot 2^x$



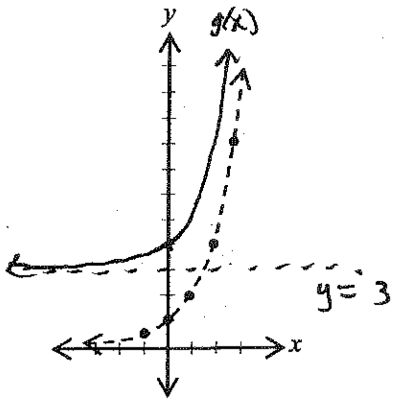
Vertical stretch by $\underline{\underline{3}}$

b) $g(x) = 2^{x-3}$



horiz. right 3

c) $g(x) = 2^x + 3$



vert up 3

The parent function for each graph below is of the form $y = ab^x$. Write the parent function. Then write a function for the translation indicated. $y = ab^x$

(0, -4)
(1, -2)
(2, -1)

$y = 2^x$

1.

(0, 1)
(1, 2)

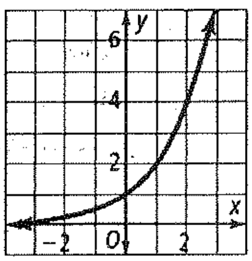
$y = ab^x$

$1 = ab^0$

$a = 1$

$2 = 1b^1$

$b = 2$



translation: left 3 units, up 1 unit

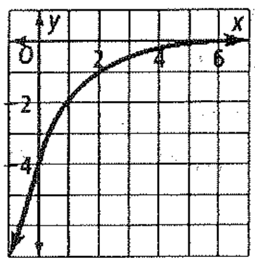
$y = 2^{x+3} + 1$

$y = ab^x$

$y = (1)(2)^x$

$y = -4\left(\frac{1}{2}\right)^x$

2.



translation: right 3 units, up 1 units

$y = -4\left(\frac{1}{2}\right)^{x-3} + 1$

Date: ______________

Notes 7.3: Log Functions as Inverses

To solve the exponential equation: $2^x = 16$

rewrite 16 as a power of 2: $2^x = 2^4$

Same base $\Rightarrow$ exp. are =: $x = 4$

What about the exponential equation: $2^x = 10$?

To solve this equation, we need logarithms.

Note: log = exponent

Every exponential equation can be written in log form. Ex: $2^4 = 16 \Leftrightarrow 4 = \log_2 16$

Equivalent Exponential and Log Forms: For any positive base, b, $b \neq 1$: $b^x = y \Leftrightarrow x = \log_b y$

[$\text{base}^{\text{exp}} = \# \longleftrightarrow \text{exp} = \log_{\text{base}} \#$]

Write each equation in logarithmic form. [Hint: start w/ the exponent.]

a) $1000 = 10^3 \Leftrightarrow 3 = \log_{10} 1000$

b) $81^{\frac{1}{4}} = 3 \Leftrightarrow \frac{1}{4} = \log_{81} 3$

c) $\frac{1}{25} = 5^{-2} \Leftrightarrow -2 = \log_5 \frac{1}{25}$

Write each equation in exponential form. [Hint: start w/ the base.]

a) $\log_2 32 = 5 \Leftrightarrow 2^5 = 32$

b) $\log_{144} 12 = \frac{1}{2} \Leftrightarrow 144^{\frac{1}{2}} = 12$

c) $-3 = \log_3 \frac{1}{27} \Leftrightarrow 3^{-3} = \frac{1}{27}$

When the **base** of a log is 10, the log is called the common log. Notation: $\log_{10} 5 = \log 5$

Use your calc. to compute (nearest 100th):

a) $\log_{10} 12 \approx 1.08$

b) $\log 0.5 \approx -0.30$

Solve for x (nearest 100th). Hint: Convert to log form.

a) $10^x = 31 \Leftrightarrow x = \log_{10} 31$

$x \approx 1.49$

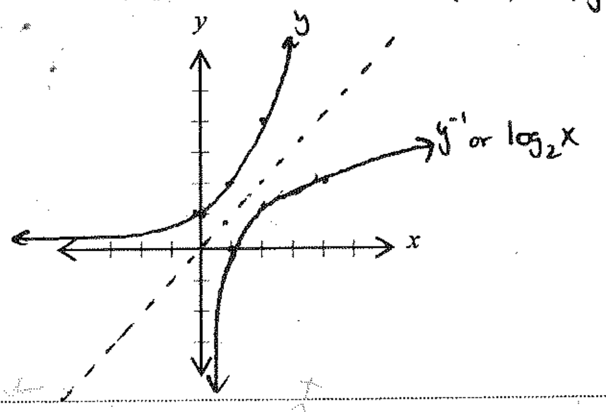
b) $10^x = \frac{3}{49} \Leftrightarrow x = \log_{10}(3/49)$

$x \approx -1.21$

Graph $y = 2^x$.

x	y
−2	1/4
−1	1/2
0	1
1	2
2	4

Then graph the inverse of $y = 2^x$. $(x, y) \rightarrow (y, x)$



Find the equation for the inverse:

$y = 2^x$ $\quad x = 2^y$

$y = \log_2 x$

Function	Domain	Range
$y = 2^x$	$\{x : x \in \mathbb{R}\}$	$\{y : y > 0\}$
$y = \log_2 x$	$\{x : x > 0\}$	$\{y : y \in \mathbb{R}\}$

Graph $y = \log_3 x$

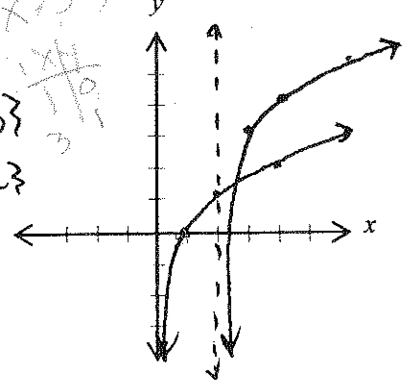
Give the:

Domain: $\{x : x > 0\}$

Range: $\{y : y \in \mathbb{R}\}$

x-intercept: (1, 0)

asymptote: $x = 0$



Using color, graph $y = \log_3(x-2)+3$ on the same axes.

List the transformations from the parent graph:

rt 2, up 3

Give the new domain and range:

D: $\{x : x > 2\}$

R: $\{y : y \in \mathbb{R}\}$

Date: ____________ Notes 7.4: ____________

Use your calc. to fill out the chart. Round number to 4 decimal places. Draw conclusions about relationships between the columns to discover the properties of logs.

n	A	B	$\log A$	$\log B$	$\log(A\cdot B)$	$\log\frac{A}{B}$	$\log A^n$	$\log A+\log B$	$\log A-\log B$	$n\cdot\log A$
2	7	9	0.8451	0.9542	1.7993	-0.1091	1.6902	1.7993	-0.1091	1.6902
3	4	7	0.6021	0.8451	1.4472	-0.2430	1.8062	1.4472	0.2430	6
6	10	5	1	0.6990	1.6990	0.3010	6	1.6990	0.3010	0
2	1	4	0	0.6021	0.6021	-0.6021	0	0.6021	-0.6021	0
4	6	0	0.7782	—	—	—	3.1126	—	—	3.1126
5	-3	9	—	0.9542	—	—	—	—	—	—

Observations:

1. $\log(A\cdot B) =$ log A + log B

   Ex: $\log(7\cdot 9) =$ log 7 + log 9

2. $\log\left(\frac{A}{B}\right) =$ log A − log B

   Ex: $\log\left(\frac{4}{7}\right) =$ log 4 − log 7

3. $\log A^n =$ n log A

   Ex: $\log 5^6 =$ 6 log 5

4. $\log_{10} 10 = 1$

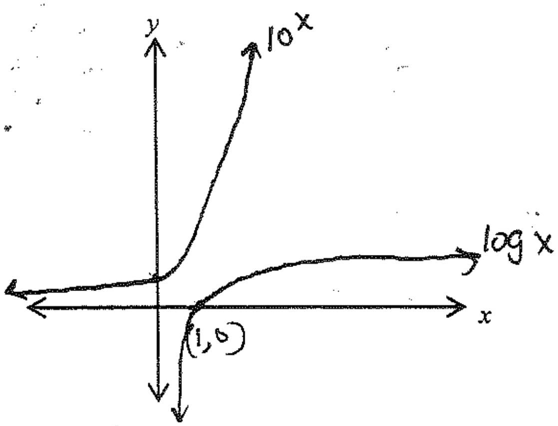
5. $\log_{10} 1 = 0$

6.

log 0 (or neg #) = ∅

Review. Sketch $f(x) = \log x$.

[Hint: start w/ $f(x) = 10^x$.]



D = {x : x > 0}

R = {y : y ∈ ℝ}

Properties of Log Functions ($A > 0$, $B > 0$, $b > 0$, $b \neq 1$)

1. Product Property: $\log_b(A\cdot B) = \log_b A + \log_b B$

2. Quotient Property: $\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B$

3. Power Property: $\log_b A^n = n\log_b A$

Note: $\log_b(A+B) \neq \log_b A \cdot \log_b B$

$\frac{\log_b A}{\log_b B} \neq \log_b(A - B)$

Doesn't work Backward!

Product Property of Logs

Ex: $\log 15 = \log(\underline{5} \cdot \underline{3}) = \log \underline{5} + \log \underline{3}$

$\log 2 + \log 3 = \log(\underline{2} \cdot \underline{3}) = \log \underline{6}$

Proof:

Let $\log_b A = m$ (write in exp form) $\Rightarrow A = b^m$

$\log_b B = n$ (write in exp form) $\Rightarrow B = b^n$

then $A \cdot B = b^m \cdot b^n$

$A \cdot B = b^{m+n}$

Write in *log* form: $\log_b(A \cdot B) = m + n$

Subst. *m* and *n*: $\boxed{\log_b(A \cdot B) = \log_b A + \log_b B}$

Quotient Property of Logs

Ex: $\log \frac{3}{2} = \log \underline{3} - \log \underline{2}$

$\log 12 - \log 3 = \log\left(\frac{12}{3}\right) = \log \underline{4}$

Proof:

Let $\log_b A = m \Rightarrow A = b^m$

$\log_b B = n \Rightarrow B = b^n$

then $\frac{A}{B} = \frac{b^m}{b^n}$

$\frac{A}{B} = b^{m-n}$

Write in *log* form: $\log_b(A/B) = m - n$

Subst. *m* and *n*: $\boxed{\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B}$

Power Property of Logs

Expand and rewrite in a different form.

$\log_b a^3 = \log_b(\underline{a} \cdot \underline{a} \cdot \underline{a})$

$= \log_b \underline{a} + \log_b a + \log_b a$

$= 3 \log_b a$

In general:
$\log_b A^n = n \log_b A$

Use the properties of logs to write as a *single* log expression. Then simplify, if possible.

a) $\log_6 18 + \log_6 2$

$\log_6(18 \cdot 2)$

$\log_6 36$

* $\log_6 6^2$

$= \boxed{2}$

b) $\log_b u + \log_b v - \log_b uw$

$\log_b uv - \log_b uw$

$\log_b \frac{\cancel{u}v}{\cancel{u}w}$

$\boxed{\log_b \frac{v}{w}}$

c) $\log_5 3x - 2\log_5 7$

$\log_5 \frac{3x}{7^2}$

$= \boxed{\log_5 \frac{3x}{49}}$

* d) $5\log_2 x - \log_2 y - 1$

$\log_2 x^5 - \log_2 y - \log_2 2$

$\log_2 \frac{x^5}{y} - \log_2 2$

$\boxed{\log_2 \frac{x^5}{2y}}$

Use the given values to evaluate the following. Given: $\log_3 6 \approx 1.6309$, $\log_3 7 \approx 1.7712$. *Note*: you also know $\log_3 3 = \underline{1}$

[Hint: Rewrite the given number as a product, quotient, or power of the numbers $\underline{3}$, $\underline{6}$, and $\underline{7}$. Then use the prop. of logs.]

a) $\log_3 42 = \log_3(6 \cdot 7)$

$= \log_3 6 + \log_3 7$

$\approx 1.6309 + 1.7712$

$\approx \boxed{3.4021}$

b) $\log_3 49 = \log_3 7^2$

$= 2(\log_3 7)$

$\approx 2(1.7712)$

$\approx \boxed{3.5424}$

c) $\log_3 \frac{18}{7} = \log_3(3 \cdot 6) + \log_3(7)$

$= \log_3 3 + \log_3 6 + \log_3 7$

$\approx 1 + 1.6309 + 1.7712$

$\approx \boxed{0.8597}$

Special cases – Use your calculator to evaluate.

$\log_{10} 10^2 = 2$

$\log_{10} 10^3 = 3$

$\log_{10} 10000 = \log_{10} 10^{4} = 4$

$\log_{10} \frac{1}{100} = \log_{10} 10^{-2} = -2$

> In general:
>
> $\log_b b^x = x$

$10^{\log_{10} 23} = 23$

$10^{\log_{10} 7} = 7$

$10^{\log_{10} 1.9} = 1.9$

$10^{\log_{10} x} = x$

> In general:
>
> $b^{\log_b x} = x$

$\log_{10} 1 = 0 \rightarrow 10^0 = 1$

$\log_2 1 = 0 \rightarrow 2^0 = 1$

$\log_3 1 = 0 \rightarrow 3^0 = 1$

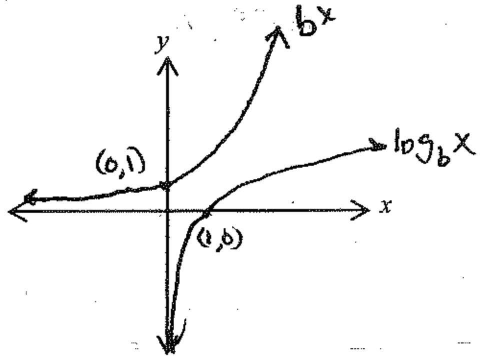
> In general:
>
> $\log_b 1 = 0$

Evaluate each expression:

1. $5^{\log_5 9} = 9$
2. $\log_7 7^{10} = 10$
3. $\log_8 1 = 0$
4. $\log_2 \frac{1}{8} = \log_2 2^{-3}$ $= \boxed{-3}$
5. $\log_2(\log_3 81) = \log_2(\log_3 3^4)$ $= \log_2(4)$ $= \log_2 2^2 = \boxed{2}$
6. $\log_4 \frac{1}{16} - 6^{\log_6 12} + \log_3 3^5 - 7\log_2 1 =$ $\log_4 4^{-2} - 12 + 5 - 7(0) =$ $-2 - 12 + 5 = \boxed{-9}$

Another look at *special cases.*

Review: On the axes, sketch the graphs of $f(x) = b^x$ and $g(x) = \log_b x$



Draw the line $y = x$. What is true about these two functions?

List the *special* property of functions and their inverses.

$f(g(x)) = g(f(x)) = x$

Use that property to *derive* the two special cases.

$f(x) = b^x$ and $g(x) = \log_b x$

$f(g(x)) = x$	$g(f(x)) = x$
$f(\log_b x) = x$	$g(b^x) = x$
$b^{\log_b x} = x$	$\log_b b^x = \cancel{x}$

Apply:

a) $7^{\log_7 11} = \boxed{11}$ b) $\log_5 5^2 = \boxed{2}$ c) $\log_2 16 =$ $\log_2 2^4 = \boxed{4}$ d) $\log_5 25^4 = \boxed{4}$ e) $\log_3 27^{100} =$ $\log_3 (3^3)^{100}$ $\boxed{300}$

Note: We will derive this in section 7.5

> **The Change-of-Base Formula**
>
> $\log_b x = \dfrac{\log x}{\log b}$ or
>
> $\log_b x = \dfrac{\ln x}{\ln b}$
>
> Note: *base* to the bottom (basement)

Evaluate using common logs and the change of base formula:

1. $\log_4 18 = \dfrac{\log 18}{\log 4}$ $\approx \boxed{2.08}$

2. $\log_{18} 4 = \dfrac{\log 4}{\log 18} \approx \boxed{.4796}$

Date ____________ Notes 7.5: Exponential & Log Functions

Solve for x. [Hint: write in exp. form.]

a) $\log_3 x = 4$

$3^4 = x$

$\boxed{81 = x}$

b) $-\frac{3}{2} = \log_{16} x$

$16^{-3/2} = x$

$\frac{1}{(\sqrt{16})^3} = x$

$x = \frac{1}{4^3}$

$\boxed{x = \frac{1}{64}}$

c) $\log_x 5 = \frac{1}{3}$

$(x^{1/3})^3 = (5)^3$

$\boxed{x = 125}$

$a^x = a^y$ if and only if $x = y$. Try to rewrite both sides as powers of the original base.

d) $x = \log_4 64$

$4^x = 64$

$4^x = 4^3$

$\boxed{x = 3}$

e) $\log_2 8\sqrt{2} = x$

$2^x = 8\sqrt{2}$

$2^x = 2^3\sqrt{2}$

$2^x = 2^3 \cdot 2^{1/2}$

$\boxed{x = 3\tfrac{1}{2}}$

f) $27^{3x} = 81$

$(3^3)^{3x} = 3^4$

$9x = 4$

$\boxed{x = 4/9}$

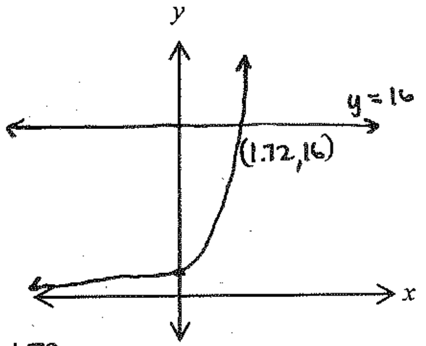
Solve: $5^x = 16$

Graphical approach.

Let $y_1 = 5^x$

$y_2 = 16$

$x \approx 1.72$



[Check your ans.] $5^{1.72} \approx 16.008$

Algebra approach.

$5^x = 16$

log log

$\log 5^x = \log 16$

$x \log 5 = \log 16$

$\frac{x \log 5}{\log 5} = \frac{\log 16}{\log 5}$

$x = \frac{\log 16}{\log 5} \approx \boxed{1.723}$

Summarize the steps:

1. Take the log of both sides.
2. Use the power property to pull down the exponent.
3. Solve for x.

Problems - Solve algebraically (use logs). [Hint: isolate the base and exp.. Then take the log of both sides.]

1. $8 + 3^x = 10$

$-8 \quad -8$

$3^x = 2$

$\log 3^x = \log 2$

$x \log 3 = \log 2$

$x = \frac{\log 2}{\log 3}$

$\approx \boxed{0.631}$

2. $8^{2x-1} = 729$

$(2x-1)\log 8 = \log 729$

$2x - 1 = \frac{\log 729}{\log 8}$

$2x = \frac{\log 729}{\log 8} + 1$

$x = \frac{1}{2}\left(\frac{\log 729}{\log 8} + 1\right)$

$\approx \boxed{2.085}$

3. $2(1.5)^x = 10$

$1.5^x = 5$

$x \log 1.5 = \log 5$

$x = \frac{\log 5}{\log 1.5}$

$\approx \boxed{3.969}$

Evaluate – Hint: Convert to exponential form first.

1. $\log_8 97 = x$

$8^x = 97$

$x \log 8 = \log 97$

$x = \dfrac{\log 97}{\log 8} \approx \boxed{2.200}$

2. $\log_b a = x$

$b^x = a$

$x \log b = \log a$

$x = \dfrac{\log a}{\log b}$

Conclusion:

The Change-of-Base Formula

$\log_b a = \dfrac{\log a}{\log b}$ or

$\log_b a = \dfrac{\ln a}{\ln b}$

Note: *base* to the bottom

Problems – Use the change of base formula to evaluate.

1. Write $\log_9 27$ in base 3

$\log_9 27 = \dfrac{\log_3 27}{\log_3 9}$

$= \dfrac{\log_3 3^3}{\log_3 3^2}$

$= \boxed{\dfrac{3}{2}}$

2. Evaluate to nearest 100^{th}: $\log_6 0.32$

$\log_6 0.32 = \dfrac{\log .32}{\log 6} \approx \boxed{-0.636}$

Solve. Check your answers. [Hint: rewrite each side as one log.] *Note*: the domain of $y = \log_b x$ is $D = \{x: x>0\}$

1. $\log_b 5 + \log_b(x-3) = \log_b(2x-3)$

$\log_b 5(x-3) = \log_b(2x-3)$

$5(x-3) = 2x-3$

$5x-15 = 2x-3$

$3x = 12$

$\boxed{x=4}$

2. $\log_5(3-t) - 2\log_5 t = \log_5 2$

$\log_5 \dfrac{(3-t)}{t^2} = \log_5 2$

$\dfrac{3-t}{t^2} = 2$

$3-t = 2t^2$

$0 = 2t^2+t-3$

$0 = (2t+3)(t-1)$

$t = -\frac{3}{2}, 1$

$\boxed{t=1}$

3. $\log_2 x + \log_2(x-2) = 3$ ← two-methods →

$\log_2 x(x-2) = 3$

$2^3 = x^2-2x$

$0 = x^2-2x+(-8)$

$0 = (x-4)(x+2)$

$\boxed{x=4}$

$\cancel{x=-2}$

4. $\log_3(x^2-4) - \log_3 x = 1$

$\log_3\left(\dfrac{x^2-4}{x}\right) = 1$

$\log_3\left(\dfrac{x^2-4}{x}\right) = \log_3 3$

$\dfrac{x^2-4}{x} = 3$

$x^2-4 = 3x$

$x^2-3x-4 = 0$

$(x-4)(x+1) = 0$

$\boxed{x=4}$

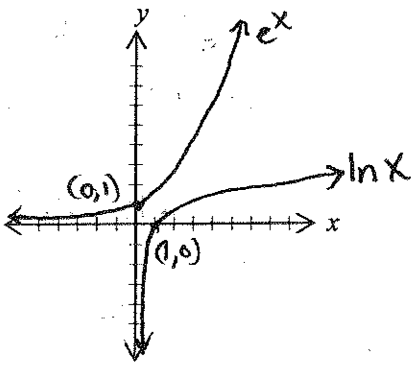
$\cancel{x=-1}$

Date ____________ Notes 7.6 Natural Logs

Connection between *natural base e exponential* functions and *natural logarithmic* functions...

$1 = \boxed{\log_e e = \ln e}$

Sketch and label the graph of $y = e^x$



Find the inverse of $y = e^x$. Sketch on the same axes (diff. color) and label.

$\ln x = \ln e^{y}$

$\ln x = y \ln e$

$\ln x = y \underline{\ln e}$

$\boxed{y = \ln x}$

Use your (calculator) TI to find the following. Round to 3 decimal places.
Then write the equivalent natural log expression or exponential expression.

$e^2 = 7.389$ $[\ln 7.389 = 2]$

$e^{-1.5} = .223$ $[\ln .223 = -1.5]$

$\ln 0.789 = -.237$ $[e^{-.237} = .789]$

$\ln 14.2 = 2.658$ $[e^{2.653} = 14.2]$

Def. of a **natural logarithm:** Always has a base $\underline{e}$.

$\ln v = u \Leftrightarrow e^u = v$

Use the properties of inverses to evaluate w/o your calculator:

1. $\ln e^{3.78} = 3.78$
2. $e^{\ln\sqrt{x+3}} = \sqrt{x+3}$

Write each expression as a single natural logarithm.

1. ln 16 – ln 8

$\ln \frac{16}{8}$

$= \boxed{\ln 2}$

2. 3 ln 3 + ln 9

$\ln 3^3 + \ln 3^2$

$\ln 3^3 \cdot 3^2$

$\ln 3^5$

$= \boxed{5 \ln 3}$

3. $\frac{1}{2}\ln a - b\ln 2$

$\ln a^{1/2} - \ln 2^b$

$\boxed{\ln\left(\frac{a^{1/2}}{2^b}\right)}$

4. 2 ln 4 – ln 8

$\ln 4^2 - \ln 2^3$

$\ln (2^2)^2 - \ln 2^3$

$\ln \frac{2^4}{2^3} = \boxed{\ln 2}$

Solve each equation. Check your answers. Round your answer to the nearest hundredth.

1. 5 ln (4x – 6) = –6

$\ln (4x-6) = \frac{-6}{5}$

$e^{-\frac{6}{5}} = 4x - 6$

$e^{-6/5} + 6 = 4x$

$\frac{e^{-6/5} + 6}{4} = x$

$\boxed{x \approx 1.58}$

2. ln x + ln 4 = 2

$\ln 4x = 2$

$e^2 = 4x$

$\frac{e^2}{4} = x$

$\boxed{x \approx 1.85}$

3. $\ln e^{x+5} = 17$

$x + 5 = 17$

$\boxed{x = 12}$

$(x+5)\ln e = 17$

$x + 5 = 17$

$e^{\ln e^{x+5}} = e^{17}$

$x + 5 = 17$

Can't take Log of Neg #

Use natural logarithms to solve each equation. Round your answer to the nearest hundredth.

1. $4e^x = 10$

$\frac{4e^x}{4} = \frac{10}{4}$

$\ln e^x = \ln 5/2$

$x = \ln 5/2$

$\boxed{x \approx .916}$

2. $e^{\frac{x}{5}} = 32$

$\ln e^{\frac{x}{5}} = \ln 32$

$\frac{x}{5} = \ln 32$

$x = 5 \ln 32$

$\boxed{x \approx 17.33}$

3. $6 - e^{12x} = 5.2$

$-6 \quad -6$

$-e^{12x} = -.8$

$\ln e^{12x} = \ln .8$

$\frac{12x}{12} = \frac{\ln(.8)}{12}$

$\boxed{x = -.019}$

4. $e^{x+6} + 5 = 1$

$-4 \quad -4$

$e^{x+6} = -4$

$\ln e^{x+6} = \ln(-4)$

$x + 6 = \cancel{\ln(-4)}$

$\boxed{\varnothing}$

Review: Solving Exponential Equations

1. Solve *without* using your calculator.

$2a^{\frac{4}{5}} = 32$

$\frac{2a^{\frac{4}{5}}}{2} = \frac{32}{2}$

$\left(a^{4/5}\right)^{5/4} = (16)^{5/4}$

$a = 2^5 = \boxed{32}$

2. Solve *without* using logs.

$\left(\frac{1}{4}\right)^{x+5} = 8$

$\left(2^{-2}\right)^{x+5} = 2^3$

$-2(x+5) = 3$

$-2x - 10 = 3$

$-2x = 13$

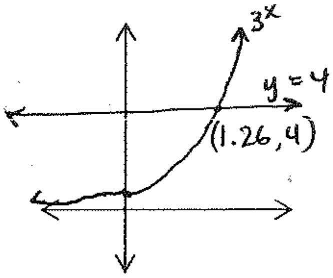
$\boxed{x = -\frac{13}{2}}$

3. Solve *graphically* with your calc. Round to 2 decimal places.

$3^x = 4$

$y_1 = 3^x$

$y_2 = 4$



$\boxed{x \approx 1.26}$

4. Solve using natural logs. Round to the nearest hundredth.

$(5.3)^x = 64$

$\ln 5.3^x = \ln 64$

$x \ln 5.3 = \ln 64$

$x = \frac{\ln 64}{\ln 5.3}$

$\boxed{x \approx 2.49}$