Date __________ Notes 8.4 Rational Expressions

Def: A **Rational Expression** is the quotient of two polynomials.

Restrictions of a rational expression: The denominator $\neq$ 0.

Example: Simplify and state the restrictions: $\dfrac{4x^2}{4x^2-8x}$

restrict from original

$$\frac{4x^2}{4x(x-2)} = \frac{\cancel{4x}\cdot x}{\cancel{4x}(x-2)} = \boxed{\frac{x}{x-2} \quad x\neq 0,2}$$

Multiply. Factor the numerator/denominator and then Cancel out the common factors.

1. $\dfrac{x^3-8}{x+4}\cdot\dfrac{3x+12}{2-x}$

$$\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x+4}}\cdot\frac{3\cancel{(x+4)}}{-1\cancel{(x-2)}} \leftarrow \text{negate}$$

$$\boxed{-3(x^2+2x+4)}$$

2. $\dfrac{x^2+x-6}{2x^2-5x+2}\cdot\dfrac{4x-8x^2}{x^2-x-12}\cdot\dfrac{x+4}{8x^2}$

$$\frac{\cancel{(x+3)}\cancel{(x-2)}}{\cancel{(2x-1)}\cancel{(x-2)}}\cdot\frac{\cancel{4x}\overset{-(2x-1)}{\cancel{(1-2x)}}}{(x-4)\cancel{(x+3)}}\cdot\frac{x+4}{\underset{2}{\cancel{8x}}\cdot x}$$

$$\boxed{-\frac{(x+4)}{2x(x-4)}}$$

Review. Divide: $\dfrac{10}{21}\div\dfrac{15}{7}=$ $\dfrac{10}{21}\cdot\dfrac{7}{15} = \dfrac{\cancel{5}\cdot 2}{\cancel{7}\cdot 3}\cdot\dfrac{\cancel{7}}{\cancel{5}\cdot 3} = \boxed{\dfrac{2}{9}}$

multiply by the reciprocal

Divide.

1. $\dfrac{x^2+2x-3}{x^2-3x+2}\div\dfrac{x^2+6x+9}{x+2}$

$$\frac{\cancel{(x+3)}\cancel{(x-1)}}{(x-2)\cancel{(x-1)}}\cdot\frac{x+2}{\cancel{(x+3)}(x+3)}$$

$$\boxed{\frac{x+2}{(x-2)(x+3)}}$$

2. $\dfrac{4-a}{5a}\div\dfrac{a^2+a-20}{a^2+5a}$

$$\frac{-\cancel{(a-4)}}{5\cancel{a}}\cdot\frac{\cancel{a}\cancel{(a+5)}}{\cancel{(a+5)}\cancel{(a-4)}} = \boxed{\frac{-1}{5}}$$

3. $\dfrac{x^2-5x+6}{x^2-8x+15}\div\boxed{\dfrac{x-2}{x-5}}\dfrac{x^2+3x}{x^2-9}$ ← reciprocate

$$\frac{\cancel{(x-3)}\cancel{(x-2)}}{\cancel{(x-5)}\cancel{(x+3)}}\cdot\frac{\cancel{x-5}}{\cancel{x-2}}\cdot\frac{x\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$$

$$\boxed{\frac{x}{x-3}}$$

Parentheses First!

4. $\dfrac{x-7}{x-4}\div\left(\dfrac{x^2-49}{3x-12}\div\dfrac{x^2+14x+49}{x+5}\right)$

$$\frac{x-7}{x-4}\div\left(\frac{(x-7)\cancel{(x+7)}}{3(x-4)}\cdot\frac{x+5}{\cancel{(x+7)}(x+7)}\right)$$

$$\frac{\cancel{x-7}}{\cancel{x-4}}\cdot\frac{3\cancel{(x-4)}(x+7)}{\cancel{(x-7)}(x+5)} = \boxed{\frac{3(x+7)}{x+5}}$$

When Dividing Rational Expressions:

1. Multiply by the reciprocal
2. Factor num/denom and cancel common factors

Date: ________ Notes 8.5 (part 1) Add / Subtract Rational Expressions

Review: To add/subtract fractions you need to get the least common denominator. [LCD]

Add: $\frac{9}{40} + \frac{7}{30} = \frac{3 \cdot 9}{3 \cdot 40} + \frac{4 \cdot 7}{4 \cdot 30} = \frac{27+28}{120} = \frac{55}{120} = \boxed{\frac{11}{24}}$

LCD 120

Determine the LCD and show the steps to create the LCD in all of the fractions. Add/subtract and simplify.

1. $\frac{5x}{x+2} - \frac{3x}{x-2}$ $\boxed{\text{LCD} = (x+2)(x-2)}$

careful

$\frac{5x\,(x-2)}{(x+2)(x-2)} - \frac{3x\,(x+2)}{(x-2)(x+2)}$

$\frac{5x^2-10x-3x^2-6x}{(x+2)(x-2)}$

$\frac{2x^2-16x}{(x+2)(x-2)} = \boxed{\frac{2x(x-8)}{(x+2)(x-2)}}$

2. $\frac{x+1}{\cancel{x^2-4}} + \frac{2}{\cancel{3x-6}}$ [Hint: 1st Factor to find the LCD.]

$(x+2)(x-2) \quad 3(x-2)$

$\boxed{\text{LCD } 3(x+2)(x-2)}$

$\frac{3(x+1)}{3(x+2)(x-2)} + \frac{2(x+2)}{3(x+2)(x-2)}$

$\frac{3x+3+2x+4}{3(x+2)(x-2)}$

$\boxed{\frac{5x+7}{3(x+2)(x-2)}}$

- Take all from 1st Denominator
- Supplement from 2nd denominator

3. $\frac{5x}{\cancel{x^2-9}} - \frac{x+1}{x+3} + \frac{2}{x-3}$ $\boxed{\text{LCD } (x+3)(x-3)}$

$(x+3)(x-3)$

$\frac{5x}{(x+3)(x-3)} - \frac{(x+1)(x-3)}{(x+3)(x-3)} + \frac{2(x+3)}{(x+3)(x-3)}$

$\frac{5x-(x^2-2x-3)+2x+6}{(x+3)(x-3)}$

$\frac{5x-x^2+2x+3+2x+6}{(x+3)(x-3)}$

$\boxed{\frac{-x^2+9x+9}{(x+3)(x-3)}}$

When Adding or Subtracting *Rational Expressions*:

1. Determine LCD, Factor if needed
2. multiply num./denom. by missing LCD factor
3. Add/Subtract numerator

Be sure to keep the denominator when you add/subtract.

Do NOT try to Solve for the variable.

* once you get the LCD, leave it alone and work in numerator

4. ~~$\frac{2}{3+\frac{5}{a}} - \frac{2}{3-\frac{5}{a}}$~~

4. ~~5.~~

$\frac{3x}{\cancel{x^2-5x+6}} - \frac{3}{\cancel{x^2-6x+9}}$

$(x-3)(x-2) \quad (x-3)(x-3)$

$\boxed{\text{LCD: } \underbrace{(x-3)(x-2)}_{\text{First denom}}\underset{\text{need another}}{(x-3)}}$

$\frac{3x(x-3)}{(x-3)(x-3)(x-2)} - \frac{3(x-2)}{(x-3)(x-3)(x-2)}$

$\frac{3x^2-9x-3x+6}{(x-3)(x-3)(x-2)}$

$\frac{3x^2-12x+6}{(x-3)(x-3)(x-2)}$

$\boxed{\frac{3(x^2-4x+2)}{(x-3)(x-3)(x-2)}}$

Date: ________ **Notes 8.5 (part 2):** Complex Fractions (Fraction within a Fraction)

Review: Simplifying Complex fractions: $\dfrac{\frac{3}{5}}{\frac{12}{35}}$ ← ÷

$$\frac{3}{5}\cdot\frac{35}{12} = \frac{\cancel{3}}{\cancel{5}}\cdot\frac{5\cdot 7}{4\cdot\cancel{3}} = \boxed{\frac{7}{4}}$$

Simplify.

1. $\dfrac{\frac{x^4-81}{3x^2+27}}{\frac{x^2-x-12}{x}}$ ← ÷

$$\frac{x^4-81}{3x^2+27}\cdot\frac{x}{x^2-x-12}$$

$$\frac{\cancel{(x^2+9)}(x^2-9)}{3\cancel{(x^2+9)}}\cdot\frac{x}{(x+4)(x-3)}$$

$$\frac{(x+3)\cancel{(x-3)}}{3}\cdot\frac{x}{(x+4)\cancel{(x-3)}} = \boxed{\frac{x(x+3)}{3(x+4)}}$$

2. $\dfrac{x-4}{x-7}\cdot\dfrac{\frac{x^2-49}{3x-12}}{\frac{x^2+14x+49}{x+5}}$ ← ÷

$$\frac{x-4}{x-7}\cdot\frac{x^2-49}{3x-12}\cdot\frac{x+5}{x^2+14x+49}$$

$$\frac{\cancel{x-4}}{\cancel{x-7}}\cdot\frac{\cancel{(x+7)}\cancel{(x-7)}}{3\cancel{(x-4)}}\cdot\frac{x+5}{\cancel{(x+7)}(x+7)}$$

$$\boxed{\frac{x+5}{3(x+7)}}$$

Multiply numerator and denominator by LCD of all denominators to Simplify.

3. LCD xy

$$\frac{\frac{1}{x}+\frac{x}{y}\,(xy)}{\frac{1}{y}+1\,(xy)}$$

$$\frac{\frac{\cancel{x}y}{\cancel{x}}+\frac{x\cdot x\cancel{y}}{\cancel{y}}}{\frac{x\cancel{y}}{\cancel{y}}+xy}$$

$$\boxed{\frac{y+x^2}{x+xy}}$$

Clears Complex!

4. LCD xy

$$\frac{3x-\frac{1}{y}\;(xy)}{\frac{y^2}{x}+x\;(xy)}$$

$$\frac{3x(xy)-\frac{1(x\cancel{y})}{\cancel{y}}}{\frac{y^2(\cancel{x}y)}{\cancel{x}}+x(xy)}$$

$$\boxed{\frac{3x^2y-x}{y^3+x^2y}}$$

Date: ________ Notes 8.6: Solving Rational Equations

Solve the following equations. $\boxed{\text{LCD } 10}$

> **Hint:**
> Multiply both sides of the equation by the LCD to eliminate all denominators.

$$\frac{x^2}{5} = \frac{3x}{10} + \frac{1}{2}$$

$$\cancel{10}^{2}\,\frac{x^2}{\cancel{5}} = \cancel{10}\cdot\frac{3x}{\cancel{10}} + \cancel{10}^{5}\cdot\frac{1}{\cancel{2}}$$

$$2x^2 = 3x + 5$$

$$2x^2 - 3x + 5 = 0$$

$$(2x-5)(x+1) = 0$$

$$\boxed{x = 5/2,\ x = -1}$$

If the denominator(s) contain Variables, you must always look for restrictions.

Review: Solve the proportion $\frac{x}{x+3} = \frac{6}{x-1}$

> BTW: When you cross-multiply it is just a short cut for multiplying both sides by the LCD to eliminate denominators.

Cross multiply

$$x(x-1) = 6(x+3)$$

$$x^2 - x = 6x + 18$$

$$x^2 - 7x - 18 = 0$$

$$(x-9)(x+2) = 0$$

$$\boxed{x = 9, -2}\ \checkmark$$

What are the restrictions on x? $x \neq$ −3 or 1 Do your answers check? yes

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Identify any restrictions on your variable. Determine the LCD and multiply both sides of the equation to eliminate all denominators. You might have to Factor the denominators first. Does your answer check?

1. $\frac{x+1}{x-1} + \frac{2}{x} =$ $\boxed{\begin{array}{c}\text{LCD } x(x-1)\\ x \neq 1, 0\end{array}}$

$$x(\cancel{x-1})\frac{x+1}{\cancel{x-1}} + \cancel{x}(x-1)\frac{2}{\cancel{x}} = x(x-1)$$

$$x(x+1) + 2(x-1) = x^2 - x$$

$$x^2 + x + 2x - 2 = x^2 - x$$

$$\cancel{x^2} + 3x - 2 - \cancel{x^2} + x = 0$$

$$4x = 2$$

$$\boxed{x = 1/2}\ \checkmark$$

2. $\frac{2x+3}{x-1} - \frac{2x-3}{x+1} = \frac{10}{x^2-1}$ $\boxed{\begin{array}{c}\text{LCD } (x+1)(x-1)\\ x \neq \pm 1\end{array}}$

$(x+1)(\cancel{x-1})$ $(\cancel{x+1})(x-1)$ $(\cancel{x+1})(\cancel{x-1})$

$$(x+1)(2x+3) - (x-1)(2x-3) = 10$$

$$2x^2 + 5x + 3 - (2x^2 - 5x + 3) = 10$$

$$\cancel{2x^2} + 5x + \cancel{3} - \cancel{2x^2} + 5x - \cancel{3} = 10$$

$$10x = 10$$

$$\boxed{\cancel{x = 1}}\quad \boxed{\varnothing}\ \text{no SOL!}$$

3. $\frac{3x}{x-1} + \frac{2x}{x-6} = \frac{4x^2 - 15x + 6}{\cancel{x^2 - 7x + 6}}$ $\boxed{\begin{array}{c}\text{LCD}\\ (x-6)(x-1)\\ x \neq 6, 1\end{array}}$

$(x-6)(x-1)$

$$(x-6)(\cancel{x-1})\frac{3x}{\cancel{x-1}} + (\cancel{x-6})(x-1)\frac{2x}{\cancel{x-6}} = \cancel{(x-6)(x-1)}\frac{4x^2-15x+6}{\cancel{(x-6)(x-1)}}$$

$$3x^2 - 18x + 2x^2 - 2x = 4x^2 - 15x + 6$$

$$5x^2 - 20x = 4x^2 - 15x + 6$$

$$5x^2 - 4x^2 - 20x + 15x - 6 = 0$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$x = \cancel{6}, -1$ ← restricted!

$$\boxed{x = -1}$$

> When solving a ***rational equation***:
> 1. Factor denominator to Find LCD
> 2. Identify restrictions
> 3. multiply both sides by LCD
> 4. Solve
>
> Watch out for extraneous solutions.

Date ________________ Notes 8.1 Inverse Variation

Review: Direct Variation

Ex: The more I study, the higher score. I receive

The longer I work, the more money. I earn.

Concept: An increase in one quantity corresponds to an increase in another

y varies directly as x $\Leftrightarrow$ $y = kx$

K: constant of variation

1. If y varies directly as x, find k if $y = 3.75$ when $x = 0.3$ and write the variation equation.

$y = kx$

$3.75 = k(.3)$

$\frac{3.75}{.3} = \frac{k(.3)}{.3}$

$\boxed{12.5 = k}$ $\boxed{y = 12.5k}$

2. If a varies directly as b and $a = 36$ when $b = -9$, find a when $b = 12$.
Two methods:

Method 1:

Step 1 $\rightarrow$ $a = kb$

Step 2 $\rightarrow$ $36 = k(-9)$

$-4 = k$

$a = -4b$ ← Step 3

$a = -4(12)$

$\boxed{a = -48}$

* does not reveal "K"

Method 2:

(Proportion) "cross-multiply"

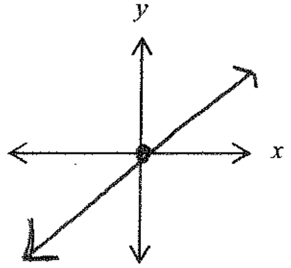
$\frac{36}{-9} = \frac{a}{12}$

$-9a = 432$

$\frac{-9a}{-9} = \frac{432}{-9}$

$\boxed{a = -48}$

3. Graph of a direct variation $y = kx$.



Two properties:

1. Passes through (0,0)
2. Slope = K (Linear)

Steps:
1) write the equation
2) substitute the given relationship to find K
3) use that K value to find the missing variable in the second relationship

Inverse Variation

Ex: The more time I spend at work, the less time I have to study

When I show all of my work, the less mistakes I make.

Concept: An increase in one quantity corresponds to a decrease in another

y varies inversely as x $\Leftrightarrow$ $y = \frac{k}{x}$

1. If y varies *inversely* as x and $y = 22.5$ when $x = 6.2$,

a) find k and write the variation equation.

$y = \frac{k}{x}$

$22.5 = \frac{k}{6.2}$

$k = 22.5(6.2)$

$\boxed{k = 139.5}$

b) find y when $x = 0.3$ (Two methods?)

$y = \frac{139.5}{.3}$

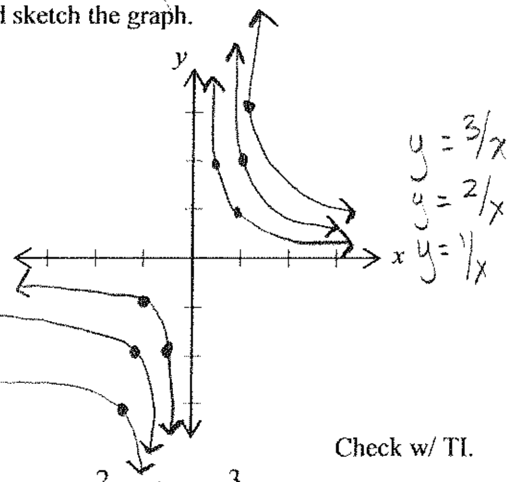
$\boxed{y = 465}$

NO, proportion doesn't work with inverse.

2. The equation $y = \frac{1}{x}$ is an example of inverse variation.

Complete the table and sketch the graph.

x	y
-3	$-1/3$
-2	$-1/2$
-1	-1
-0.5	-2
0	Undefined
0.5	2
1	1
2	$1/2$
3	$1/3$



$y = 3/x$

$y = 2/x$

$y = 1/x$

Check w/ TI.

On the same screen graph $y = \frac{2}{x}$ and $y = \frac{3}{x}$.
Use different colors and add to graph above.

The graph is called a hyperbola.

Joint Variation

y varies directly as 2 variables

Ex: Area of a $\Delta = \frac{1}{2} b \cdot h$

y varies jointly as *x* and *z* $\Leftrightarrow$ $y = kxz$

y varies jointly as *x* and *z*. If $y = -315$ when $x = 5$, $z = 9$,

a) Find *k* and write the equation.

$y = kxz$

$-315 = k(5)(9)$

$\frac{-315}{45} = \frac{45k}{45}$

$\boxed{-7 = k}$

b) Find *y* if $x = -7$, $z = 8$.

$y = -7xz$

$y = -7(-7)(8)$

$\boxed{y = 392}$

Combined Variation

A Combined variation is when you have more than 1 type of variation

z varies jointly as *x* and *y*, and inversely as *w*. If $z = 10$ when $x = 5$, $y = -2$, $w = 3$,

a) find *k* and write the equation.

$z = \frac{kxy}{w}$

$10 = \frac{k(5)(-2)}{3}$

$30 = -10k$

$\boxed{-3 = k}$

$\boxed{z = \frac{-3xy}{w}}$

b) find *z* when $x = 8$, $y = 6$, $w = -12$

$z = \frac{-3(8)(6)}{-12}$

$z = \frac{-3(\cancel{8})(\cancel{6})}{\cancel{-12}} = \boxed{12}$

Applications – Highlight the key words. Then find *k*, write the variation equation, and solve for the unknown.

1. The power in an electrical circuit varies jointly as the current and the square of the resistance. The power in a circuit is 1500 watts when the current is 15 amps and the resistance is 10 ohms. Find the power in a circuit when the current is 20 amps and the resistance is 25 ohms.

$P = kcr^2$

$1500 = k(15)(10)^2$

$1500 = 1500k$

$\boxed{1 = k}$

$P = 1(20)(25)^2$

$P = 1(20)(625)$

$\boxed{P = 1350 \text{ watts}}$

2. A baseball pitcher's earned run average (ERA) varies directly as the number of earned runs allowed and inversely as the number of innings pitched. In a recent year, a pitcher had an ERA of 2.56. He gave up 72 earned runs in 253 innings. How many earned runs would he have given up if he had pitched 300 innings, assuming that his ERA remained the same?

$ERA = k\frac{ER}{I}$

$2.56 = k \cdot \frac{72}{253}$

$(2.56)(253) = 72k$

$\frac{647.68}{72} = \frac{72k}{72}$

$\boxed{8.996 = k}$

$2.56 = \frac{8.996(ER)}{300}$

$\frac{768}{8.996} = \frac{8.996(ER)}{8.996}$

$\boxed{ER \approx 85}$

$85.37 = \text{earned runs}$